Chapter 8

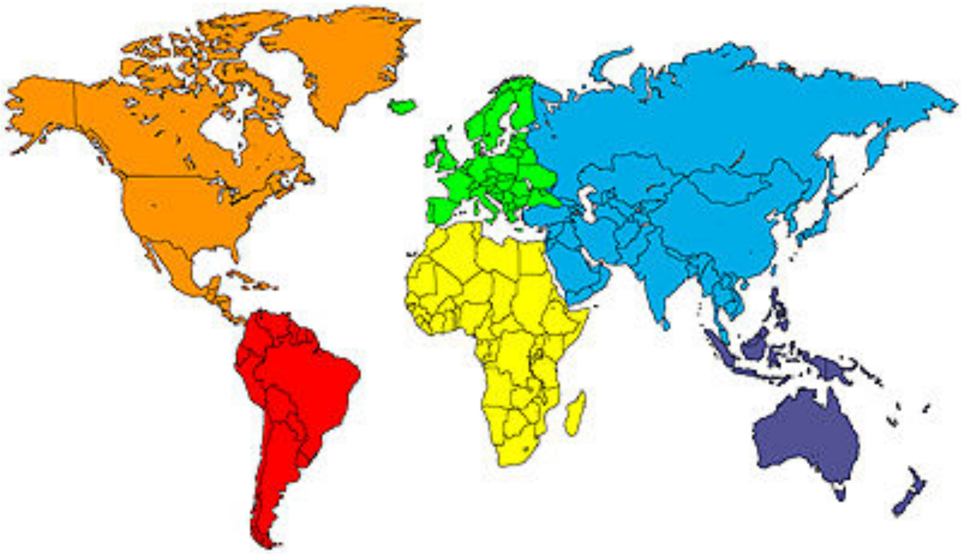
Economic Growth I: Capital Accumulation and Population Growth

IN THIS CHAPTER, YOU WILL LEARN:

- the closed economy Solow model
- how a country's standard of living depends on its _________ and ______________________.
- how to use the "Golden Rule" to find the optimal saving rate and capital stock

The World Economy

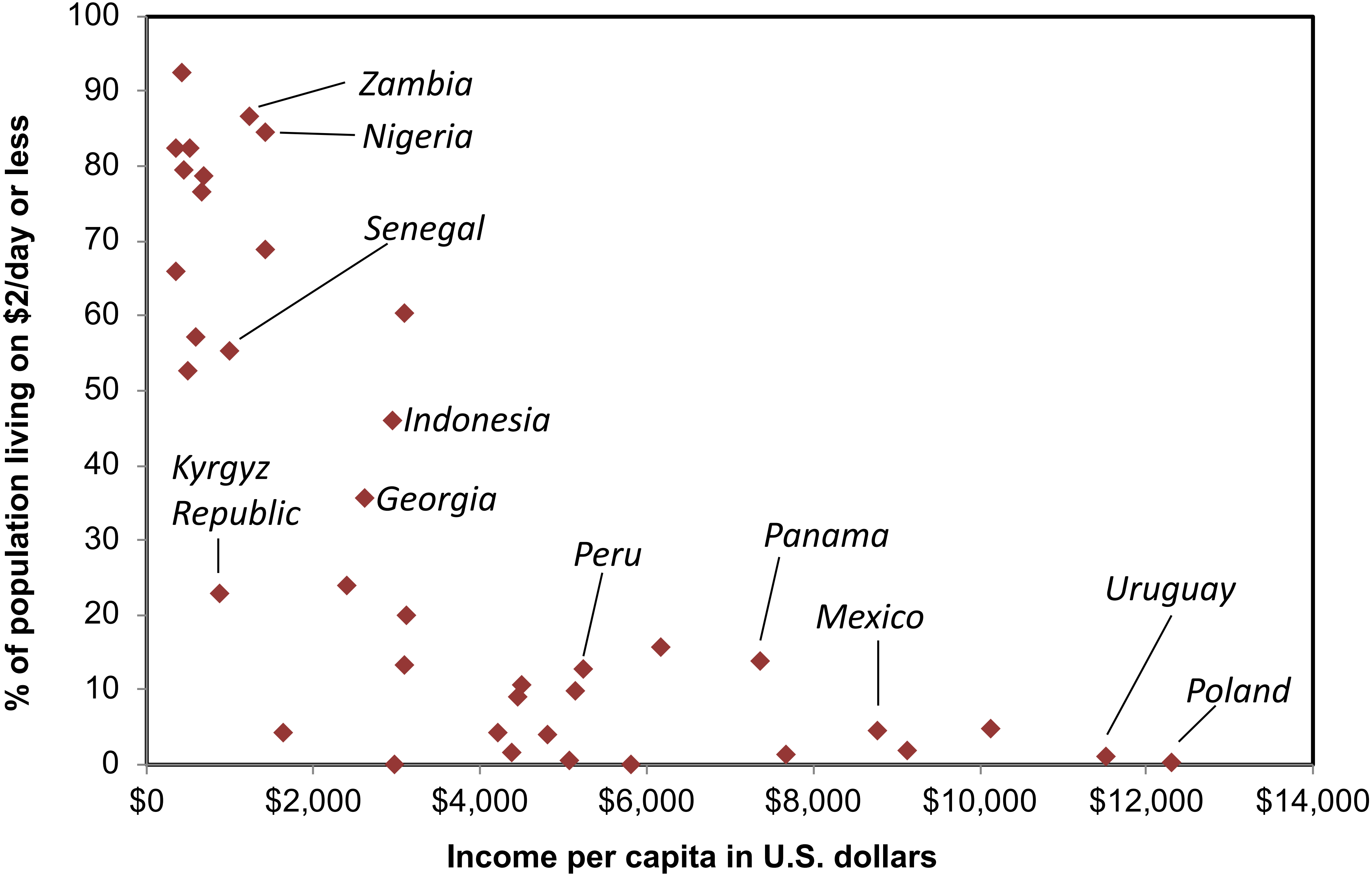
- Total GDP (2013): $87T
- Population (2013):7.1B
- GDP per Capita (2013): $13,100
- Population Growth (2013): 1.0%
- GDP Growth (2013): 2.9%



GDP per capita is probably the best measure of a country's overall well being

Income and poverty in the world

selected countries, 2010



Note. However, that growth rates vary significantly across countries/regions. Do you see a pattern here?

Region	GDP	% of World GDP	GDP Per Capita	Real GDP Growth
United States	$17T	20%	$53,000	1.6%
European Union	$16T	18%	$35,000	0.1%
Japan	$4.7T	5%	$36,300	2.0%
China	$13T	15%	$9,800	7.7%
Ghana	$90B	.1%	$3,500	7.9%
Ethiopia	$118.2B	.13%	$1,300	7.0%

Source: CIA World Factbook (2013 Estimates)

links to prepared graphs @ Gapminder.org

notes: circle size is proportional to population size,
color of circle indicates continent, press "play" on bottom to see the cross section graph evolve over time click here for one-page instruction guide

Income per capita and

Life expectancy

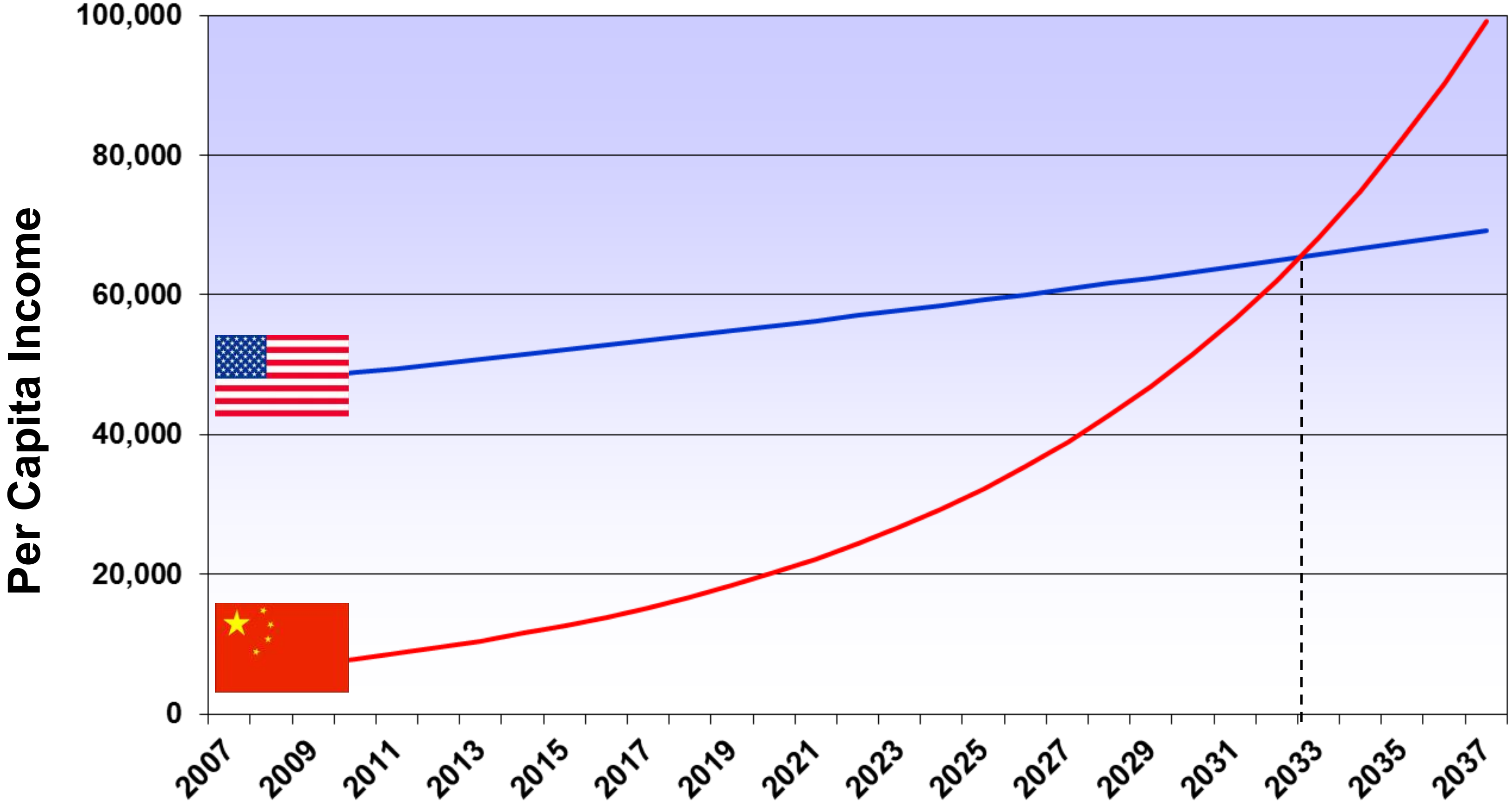
Infant mortality

Malaria deaths per 100,000

Adult literacy

Cell phone users per 100 people

At the current trends, the standard of living in China will surpass that of the US in 25 years! Or, will they?



That is, can China maintain it’s current growth rate?

As a general rule, low income (developing) countries tend to have higher average rates of growth than do high income countries

Income	GDP/Capita	GDP Growth
Low	< $1,045	6.3%
Middle	$1,045 - $12,746	4.8%
High	>$12,746	3.2%

The implication here is that eventually, poorer countries should eventually "catch up" to wealthier countries in terms of per capita income – a concept known as "convergence"

Source: World Bank (2013 estimates)

Some countries, however, don't fit the normal pattern of development

Sudan
GDP: $107B (#73)
GDP Per Capita: $5,100 (#159)
GDP Growth: -2.3% (#213)

Macau
GDP: $51.6B (#98)
GDP Per Capita: $88,700 (#3)
GDP Growth: 11.9% (#5)

At current trends, Per capita income in Macau will triple to $273,000 over the next decade. Over the same time period, per capita GDP in Sudan will drop by roughly 25%to $4,000!!!

So, what is Sudan doing wrong? (Or, what is Macau doing right?)

The Solow model

- due to Robert Solow,
  won Nobel Prize for contributions to the study of economic growth
- a major paradigm:
  - widely used in policy making
  - benchmark against which most recent growth theories are compared
- looks at the determinants of economic growth and the standard of living in the long run

How Solow model is different from Chapter 3's model

1. _____ is no longer fixed:
   investment causes it to ________,
   depreciation causes it to ________; (grow; shrink)
2. _____ is no longer fixed:
   population growth causes it to _______.
3. the consumption function is simpler

How Solow model is different from Chapter 3's model

4. no ***G*** or ***T***
   (only to simplify presentation;
   we can still do fiscal policy experiments)
5. cosmetic differences

The production function

"is a function of"

$$Y = F(K, L)$$

Real GDP

Capital Stock

Labor

Real GDP = Constant Dollar (Inflation adjusted) value of all goods and services produced in the United States

Capital Stock = Constant dollar value of private, non-residential fixed assets

Labor = Private Sector Employment

Suppose we have the following Cobb-Douglas production function:

A 1% rise in capital raises GDP by 1/3%

A 1% rise in employment raises GDP by 2/3%

$$Y = AK^{\frac{1}{3}}L^{\frac{2}{3}}$$

We can rewrite the production function in terms of growth rates to decompose GDP growth into growth of factors:

$$\underbrace{\%\Delta Y}_{\text{Real GDP Growth (observable)}} = \underbrace{(\%\Delta A)}_{\text{Productivity Growth (unobservable)}} + \frac{1}{3}\underbrace{(\%\Delta K)}_{\text{Capital Growth (observable)}} + \frac{2}{3}\underbrace{(\%\Delta L)}_{\text{Employment Growth (observable)}}$$

We are concerned with capital based growth. Therefore, growth in productivity and employment will be taken as given

$$Y = AK^{\frac{1}{3}}L^{\frac{2}{3}}$$

Productivity grows at rate

$$g_A$$

$$L = \left(\frac{L}{LF}\right)\left(\frac{LF}{Pop}\right)Pop$$

Population grows at rate

$$g_L$$

$\dfrac{\text{Employment}}{\text{Labor Force}}$ = Employment Ratio

(Assumed Constant)

$\dfrac{\text{Labor Force}}{\text{Population}}$ = Participation rate

(Assumed Constant)

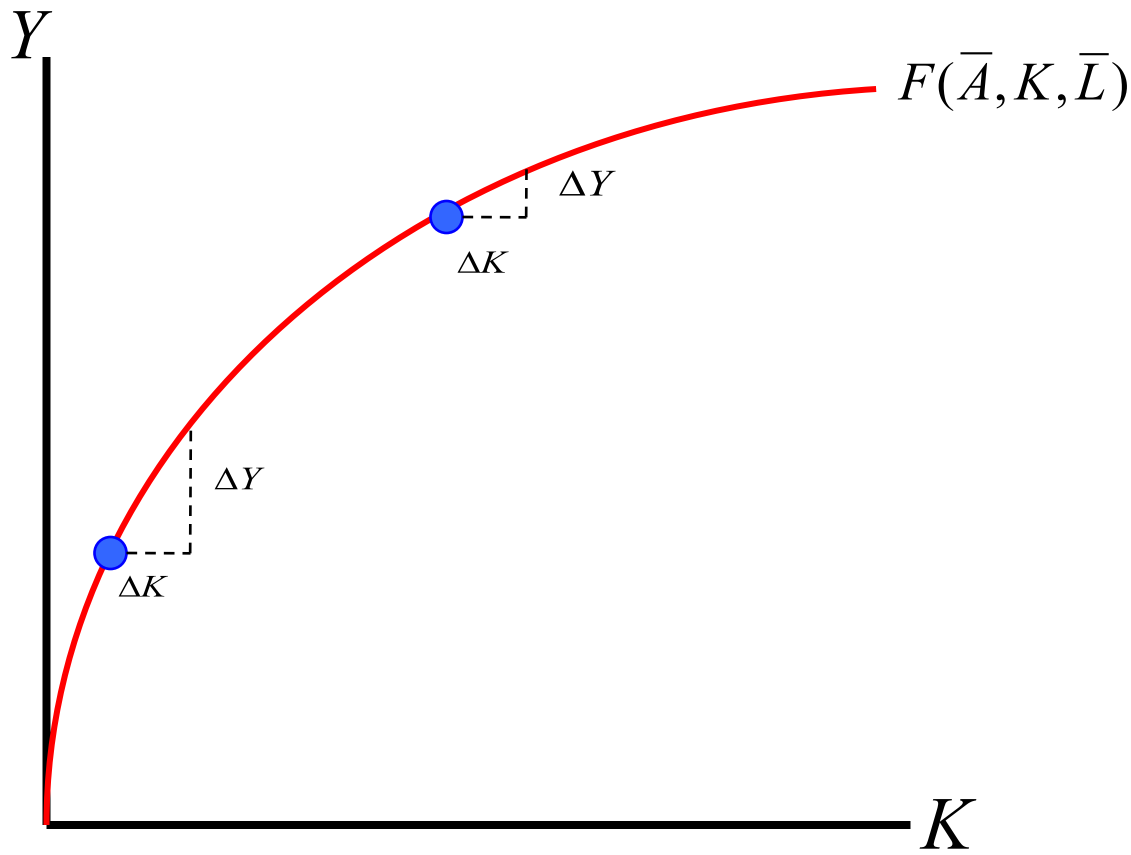
Our simple model of economic growth begins with a production function with one key property – diminishing marginal product of capital

$$Y = AK^{\frac{1}{3}}L^{\frac{2}{3}}$$

Change in Production

$$MPK = \frac{\Delta Y}{\Delta K}$$

Change in Capital Stock



As the capital stock increases (given a fixed level of employment), the productivity of capital declines!!

An economy can't grow through capital accumulation alone forever!

The national income identity

- $\boldsymbol{Y} = \boldsymbol{C} + \boldsymbol{I}$ (remember, no $\boldsymbol{G}$)
- In "per worker" terms:
- The production function:
    - has constant returns to scale in capital and labor
    - has an exponent of one-third on capital → decreasing returns to capital
  - Variables are time subscripted—they may potentially change over time
- Output can be used for either consumption (C_t) or investment (I_t): $\boldsymbol{Y_t = C_t + I_t}$

The consumption function

- ***s*** = the saving rate,
  the fraction of income that is saved
  (***s*** is an exogenous parameter)

  Note: ***s*** is the *only* lowercase variable that is *not equal to* its uppercase version divided by ***L***

- Consumption function: ???

  (per worker)

Saving and investment

- saving (per worker)

- National income identity is __________

  Rearrange to get: ____________

  (investment = saving, like in chap. 3!)

- Using the results above,

Capital Accumulation

- The capital stock next year equals the sum of the capital started with this year plus the amount of investment undertaken this year minus depreciation

- *Depreciation* is the amount of capital that wears out each period ~ 10 percent/year

$$K_{t+1} = K_t + I_t - \bar{d}K_t$$

$$\Delta K_t \equiv K_{t+1} - K_t \qquad \Delta K_t = I_t - \bar{d}K_t$$

The Model Summarized

Unknowns/endogenous variables: Y_t, K_t, L_t, C_t, I_t	
Production function	$Y_t = \bar{A}K_t^{1/3}L_t^{2/3}$
Capital accumulation	$\Delta K_t = I_t - \bar{d}K_t$
Labor force	$L_t = \bar{L}$
Resource constraint	$C_t + I_t = Y_t$
Allocation of resources	$I_t = \bar{s}Y_t$
Parameters: $\bar{A}, \bar{s}, \bar{d}, \bar{L}, \bar{K}_0$	

TABLE 5.2 The Solow Model: 5 Equations and 5 Unknowns

Solving the Solow Model

- To solve the model, write the endogenous variables as functions of the parameters of the model and graphically show what the solution looks like and solve the model in the long run.
  - combine the investment allocation equation with the capital accumulation equation

$$\Delta K_t = \bar{s}Y_t - \bar{d}K_t$$

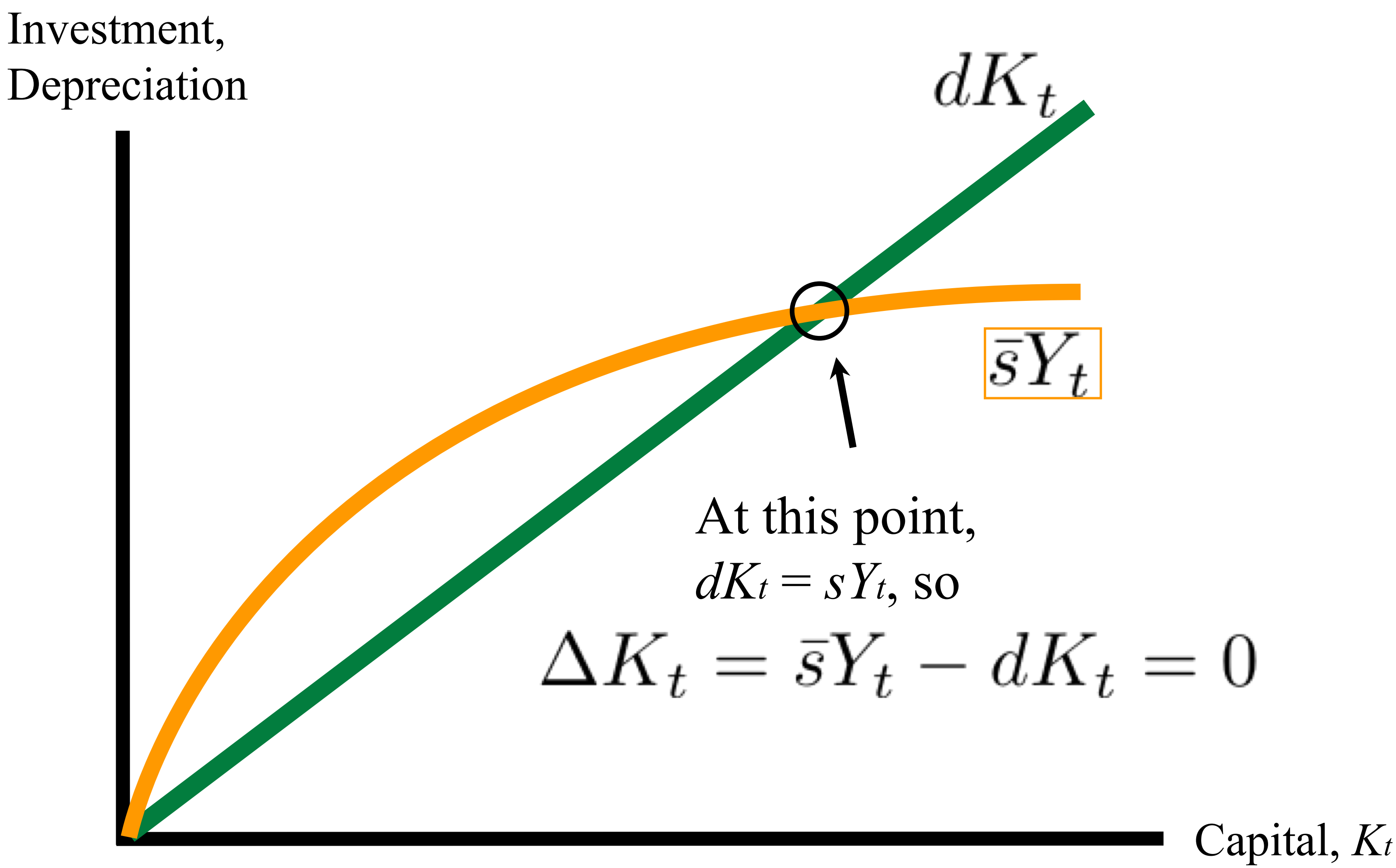
(change in capital) *(net investment)*

  - *net investment* is investment minus depreciation
  - substitute the supply of labor, $\bar{L}$, into the production function:

$$Y_t = \bar{A}K_t^{1/3}\bar{L}^{2/3}$$

  - These two equations, the capital accumulation relation and the production function, are all we need to solve the Solow model

The Solow Diagram graphs the production function and the capital accumulation relation together, with K_t on the x-axis:

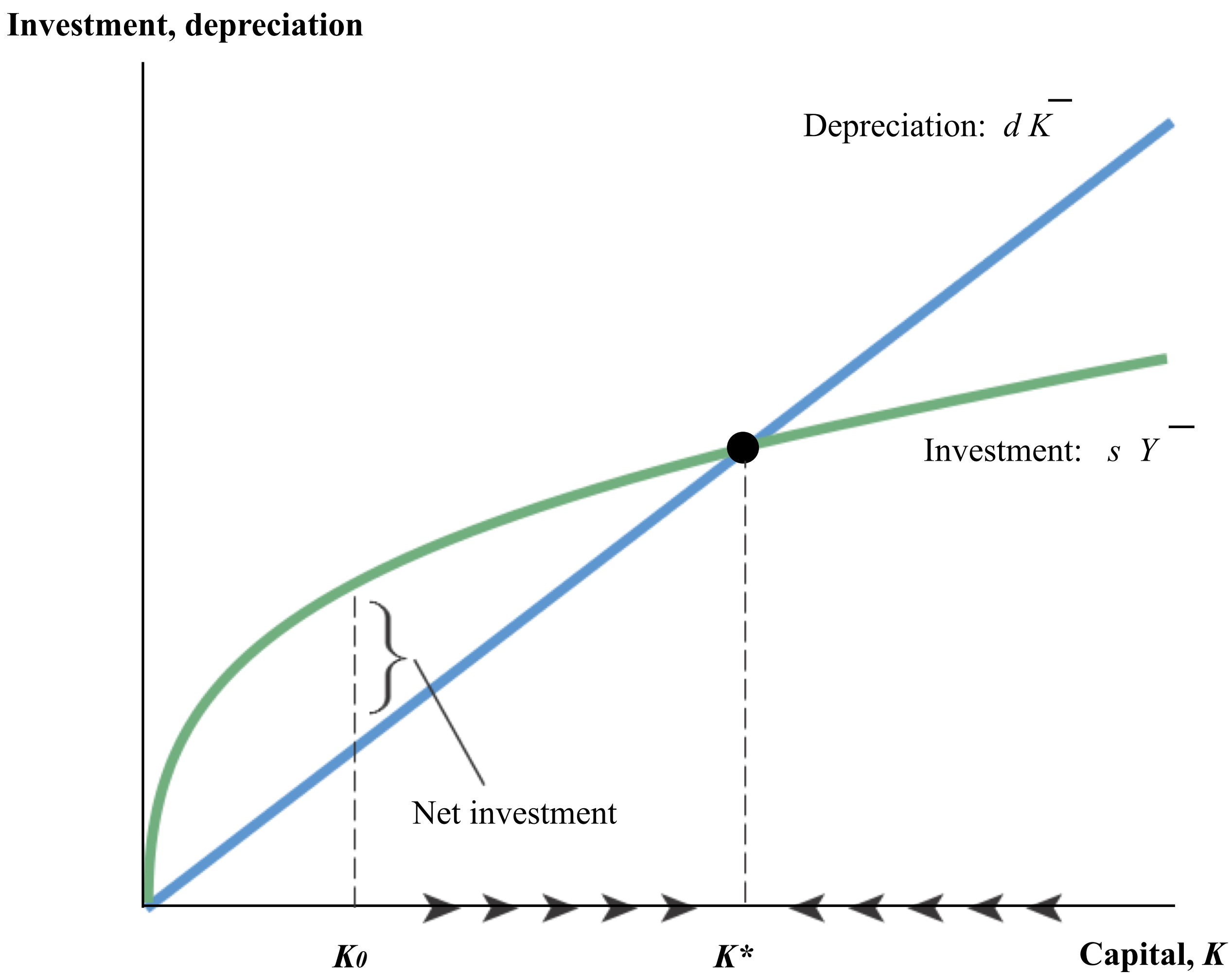


The Solow Diagram:

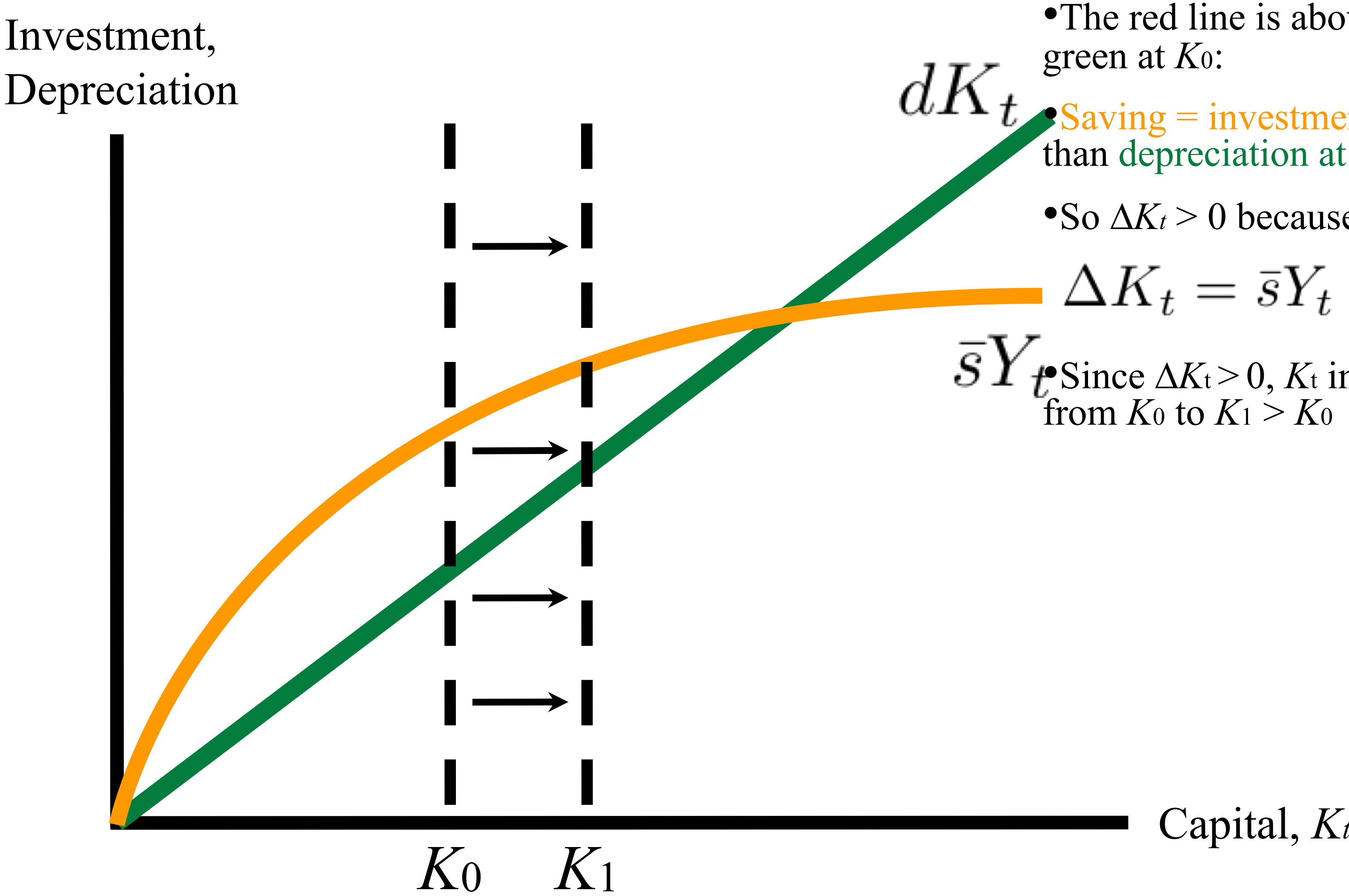
When investment is greater than depreciation, the capital stock increases

The capital stock rises until investment equals depreciation:

At this *steady state* point, *ΔK = 0*



Suppose the economy starts at K_0:

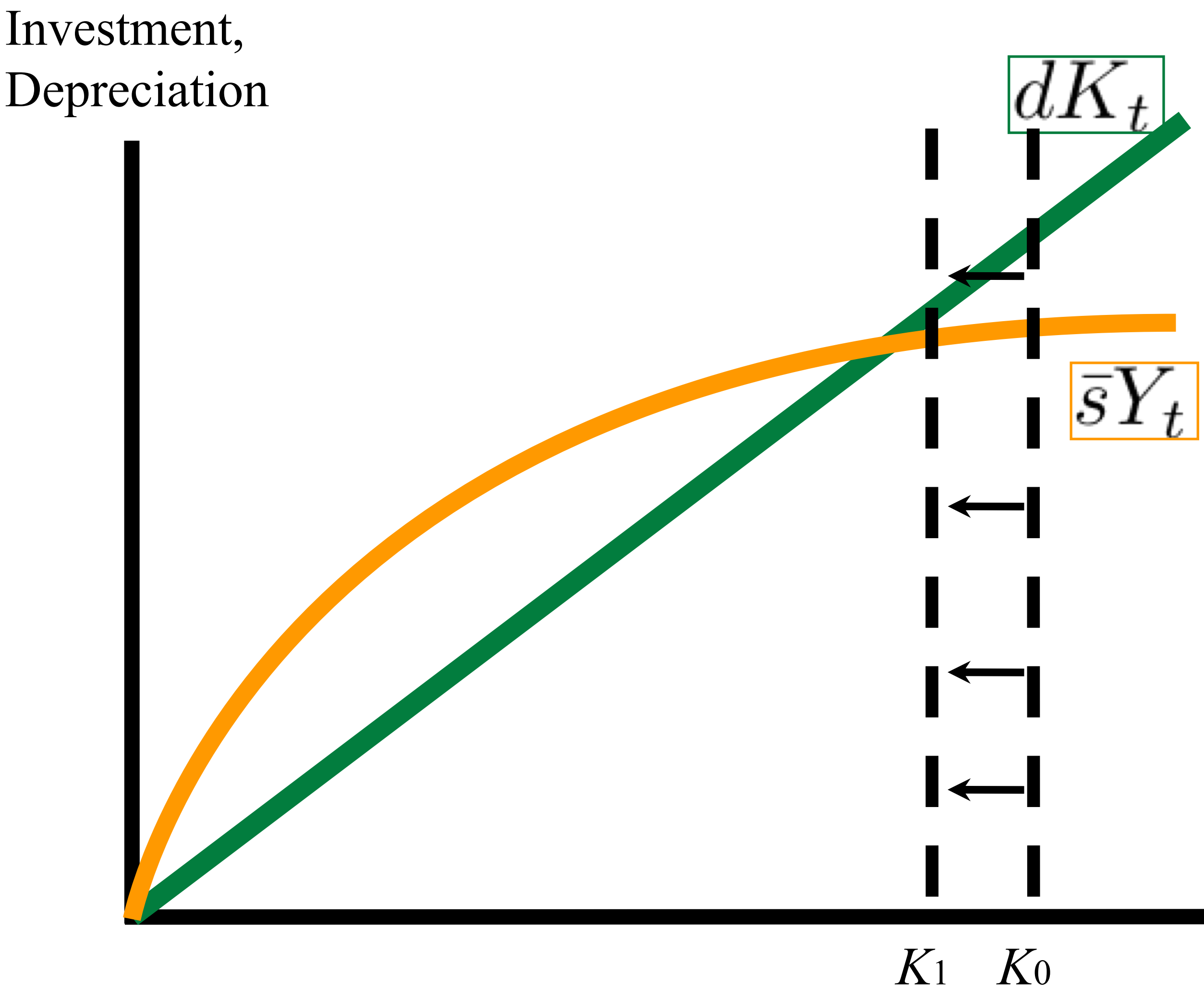


- The red line is above the green at K_0:
- Saving = investment is greater than depreciation at K_0
- So $\Delta K_t > 0$ because

$$\Delta K_t = \bar{s}Y_t - dK_t$$

- Since $\Delta K_t > 0$, K_t increases from K_0 to $K_1 > K_0$

Now imagine if we start at a K_0 here:

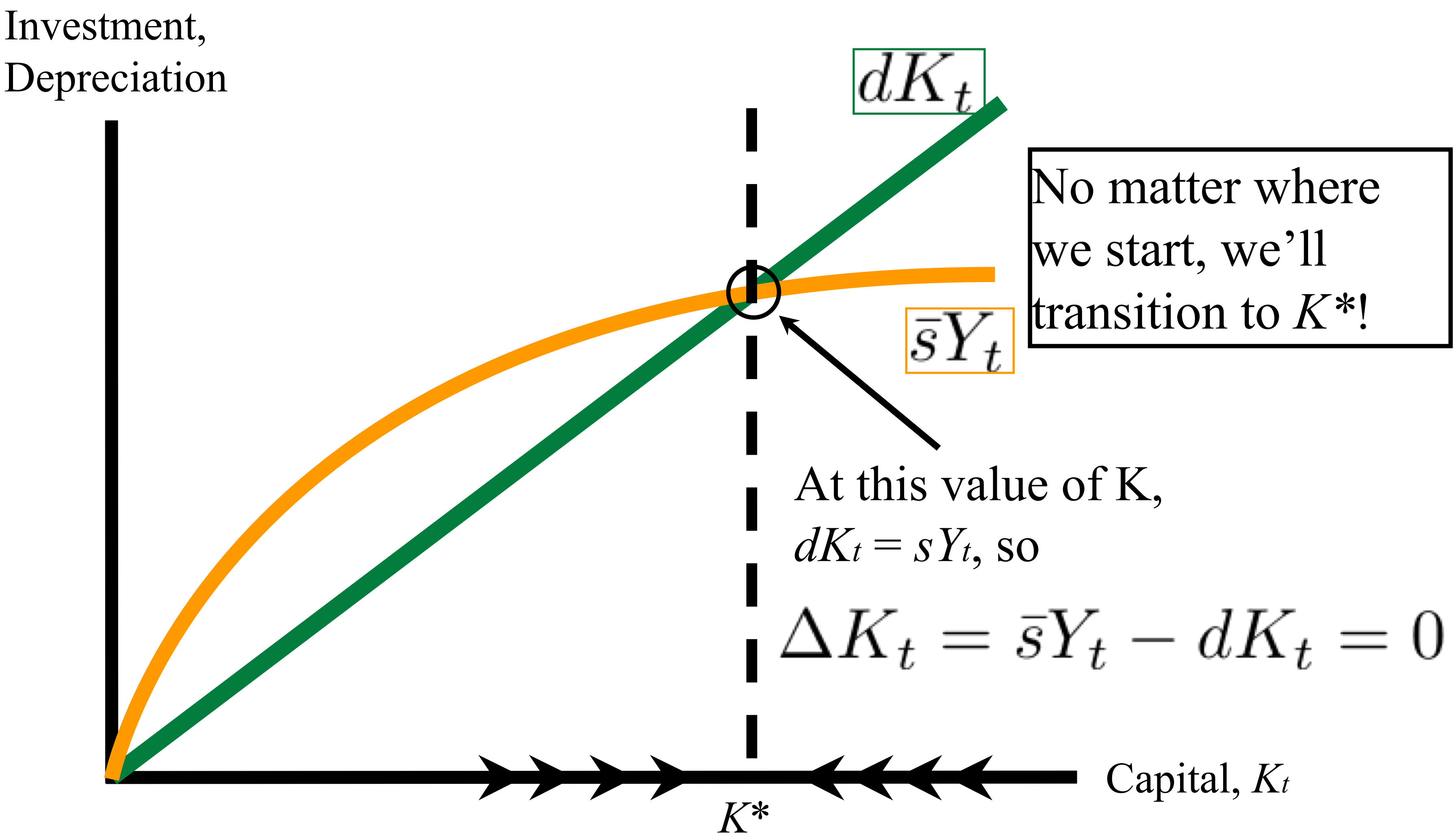


- At K_0, the green line is above the red line
- Saving = investment is now <u>less than</u> depreciation
- So $\Delta K_t < 0$ because

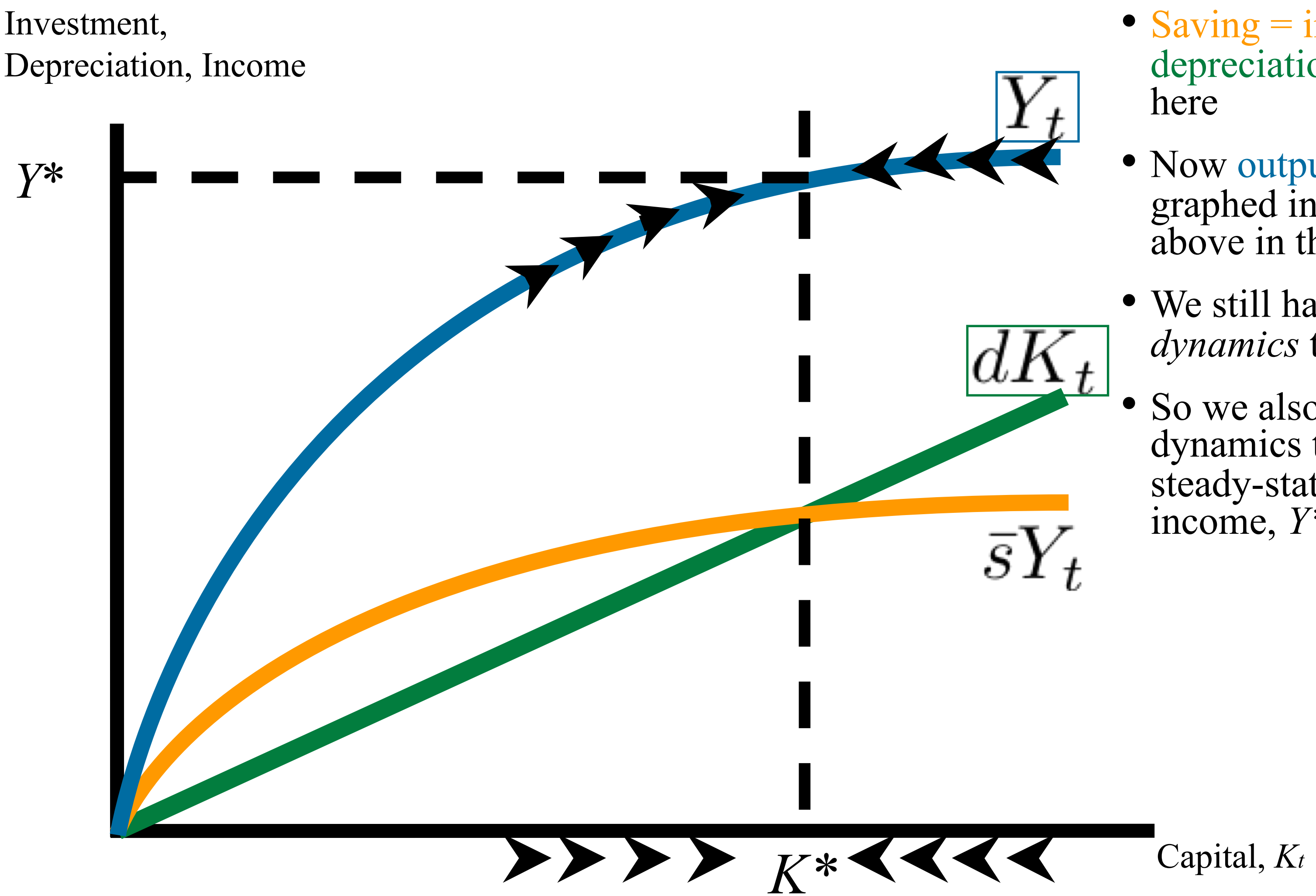
$$\Delta K_t = \bar{s}Y_t - dK_t$$

- Then since $\Delta K_t < 0$, K_t **de**creases from K_0 to $K_1 < K_0$

We call this the process of transition dynamics: Transitioning from any K_t toward the economy's steady-state K^*, where $\Delta K_t = 0$ and *growth ceases*



We can see what happens to output, *Y*, and thus to growth if we rescale the vertical axis:

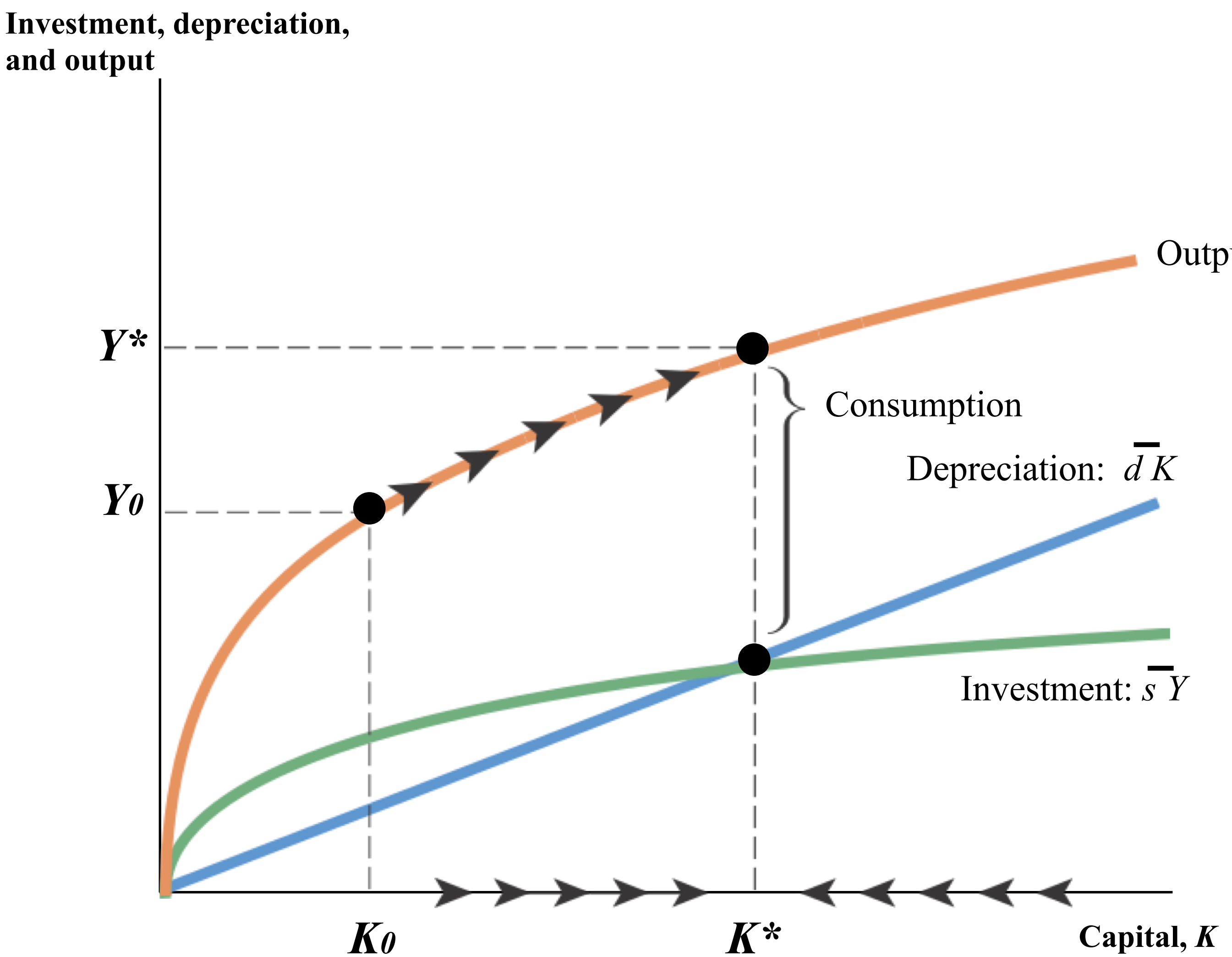


- Saving = investment and depreciation now appear here
- Now output can be graphed in the space above in the graph
- We still have *transition dynamics* toward *K**
- So we also have dynamics toward a steady-state level of income, *Y**

The Solow Diagram with Output

At any point, Consumption is the difference between Output and Investment:

$$C = Y - I$$



Solving Mathematically for the Steady State

- in the *steady state*, investment equals depreciation. If we evaluate this equation at the steady-state level of capital, we can solve mathematically for it
  - the steady-state level of capital is positively related with the investment rate, the size of the workforce and the productivity of the economy
  - the steady-state level of capital is negatively related to the depreciation rate
- In the steady state: $\Delta K_t = \bar{s}Y^* - dK^* = 0$

$$\bar{s}Y^* = dK^*$$

$$\bar{s}\bar{A}K^{*1/3}\bar{L}^{2/3} = dK^*$$

$$\bar{s}\bar{A}\bar{L}^{2/3} = d\frac{K^*}{K^{*1/3}} = dK^{*2/3}$$

$$\boxed{K^* = \left(\frac{\bar{s}\bar{A}}{d}\right)^{3/2}\bar{L}}$$

– Once we know *K**, then we can find *Y** using the production function:

$$K^* = \left(\frac{\bar{s}\bar{A}}{d}\right)^{3/2} \bar{L}$$

$$Y_t = \bar{A} K_t^{1/3} L_t^{2/3}$$

$$Y^* = \bar{A} \left(\frac{\bar{s}\bar{A}}{d}\right)^{1/2} \bar{L}^{1/3} \bar{L}^{2/3}$$

$$\boxed{Y^* = \left(\frac{\bar{s}}{d}\right)^{1/2} \bar{A}^{3/2} \bar{L}}$$

- notice that the exponent on the productivity parameter is greater than in the production function
  - higher productivity parameter raises output as in the production model.
  - higher productivity *also* implies the economy accumulates additional capital.
  - the level of the capital stock itself depends on productivity

•This solution also tells us about per capita income in the steady state, *y**, and per capita consumption as well, *c**

$$y^* \equiv \frac{Y^*}{\bar{L}} = \left(\frac{\bar{s}}{d}\right)^{1/2} \bar{A}^{3/2}$$

$$c^* = y^* - sy^* = (1 - s)\, y^*$$

Understanding the Steady State

- the economy will settle in a steady state because the investment curve, ***sY***, has diminishing returns
  - however, the rate at which production and investment rise is smaller as the capital stock is larger
  - a constant fraction of the capital stock depreciates every period, which implies depreciation is not diminishing as capital increases
- In summary, as capital increases, diminishing returns implies that production and investment increase by less and less, but depreciation increases by the same amount ***d***.
- Eventually, net investment is zero and the economy rests in steady state.
  - There are diminishing returns to capital: less Y_t per additional K_t
  - That means new investment is also diminishing: less $sY_t = I_t$
  - But depreciation is NOT diminishing; it's a constant share of K_t

Economic Growth in the Solow Model

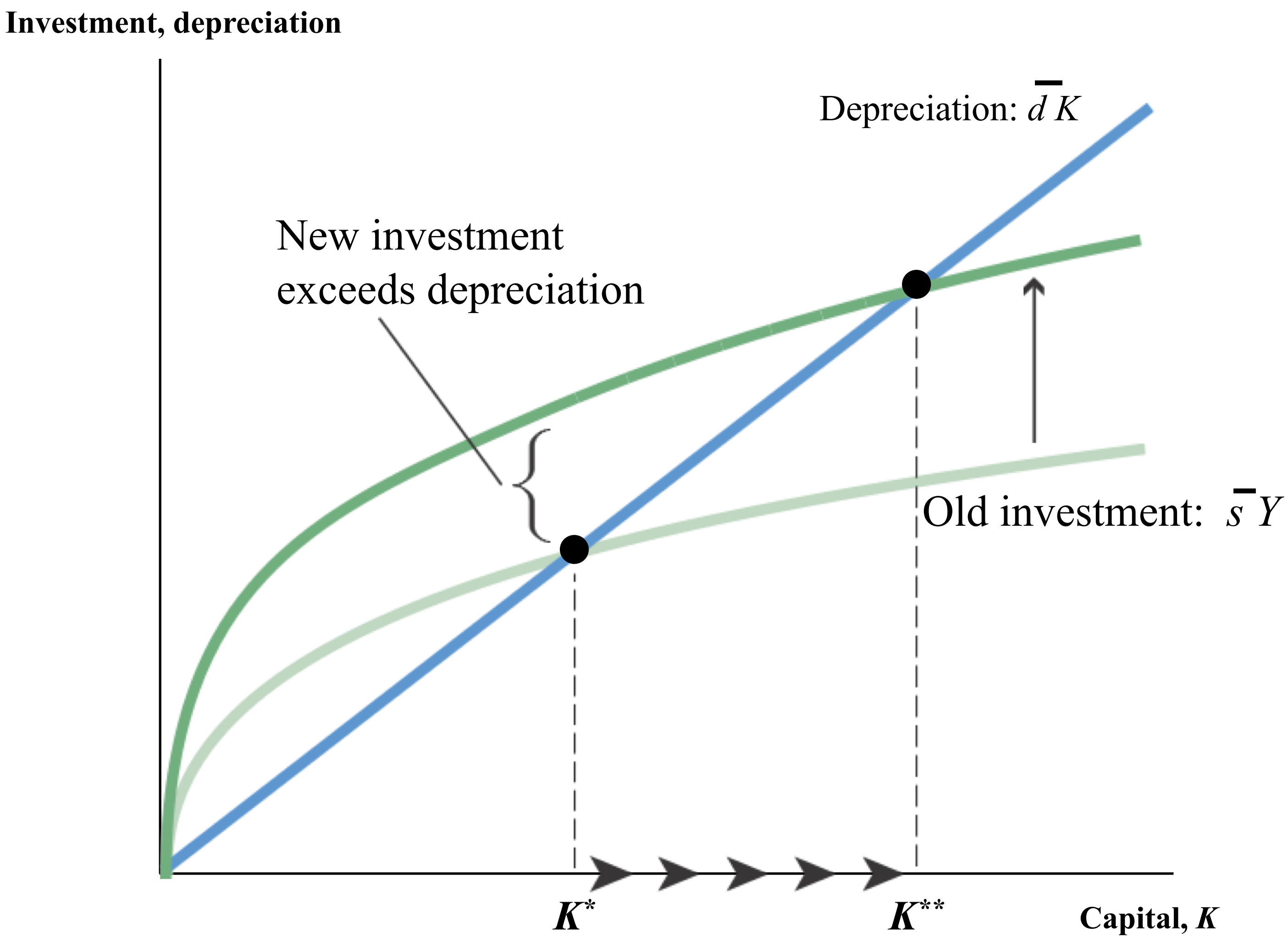
- there is no long-run economic growth in the Solow model
- in the steady state: output, capital, output per person, and consumption per person are all constant and growth stops

$$y^* \equiv Y^*/\bar{L} \qquad c^* = (1 - \bar{s})y^*$$

both constant

- empirically, economies appear to continue to grow over time
  - thus capital accumulation is not the engine of long-run economic growth
  - saving and investment are beneficial in the short-run, but diminishing returns to capital do not sustain long-run growth
  - in other words, after we reach the steady state, there is **no** long-run growth in Y_t (unless L_t or A increases)

An Increase in the Investment Rate

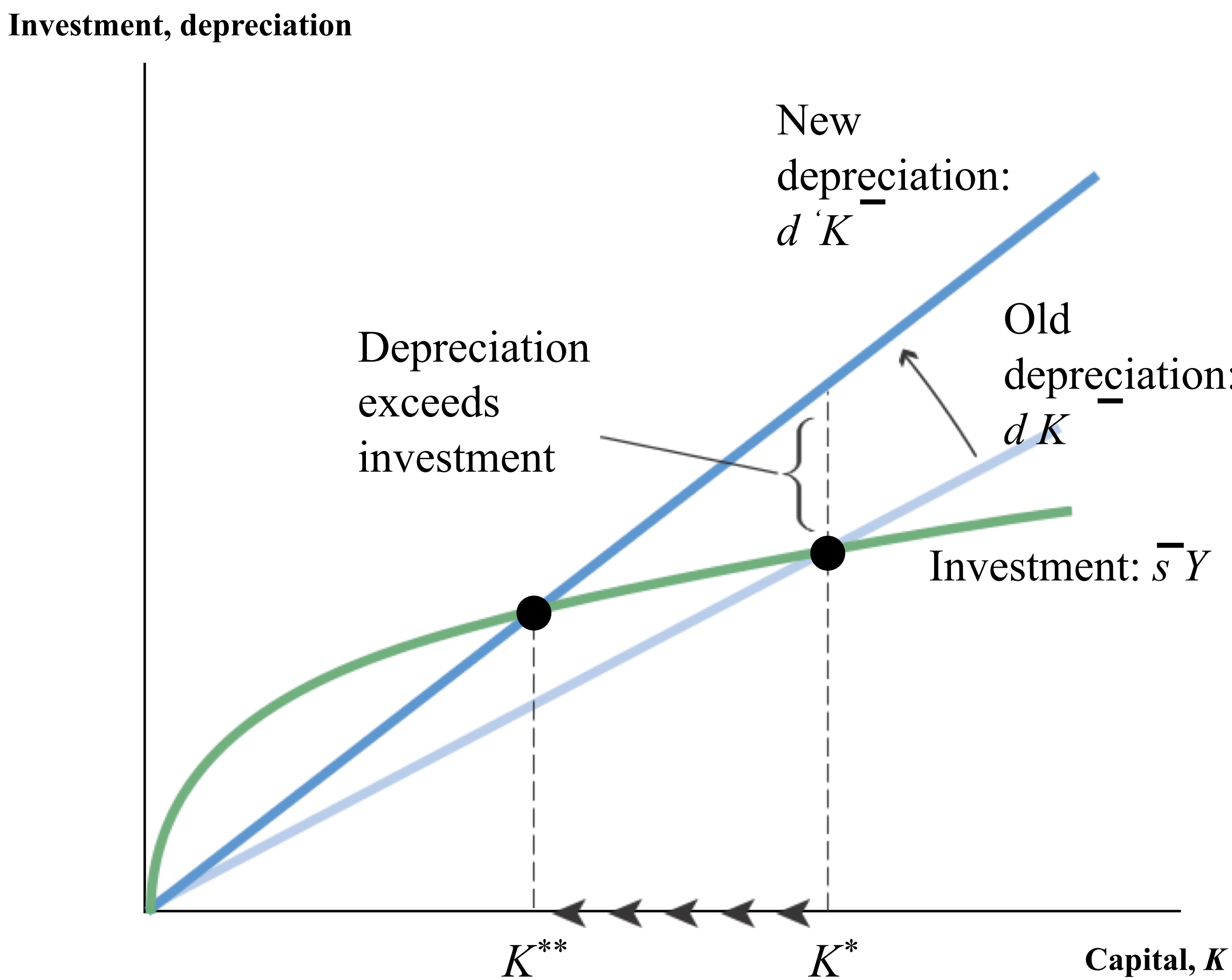


- the economy is now below its new steady state and the capital stock and output will increase over time by transition dynamics
- the long run, steady-state capital and steady-state output are higher
- What happens to output in response to this increase in the investment rate?
  - the rise in investment leads capital to accumulate over time
  - this higher capital causes output to rise as well
  - output increases from its initial steady-state level Y^* to the new steady state Y^{**}

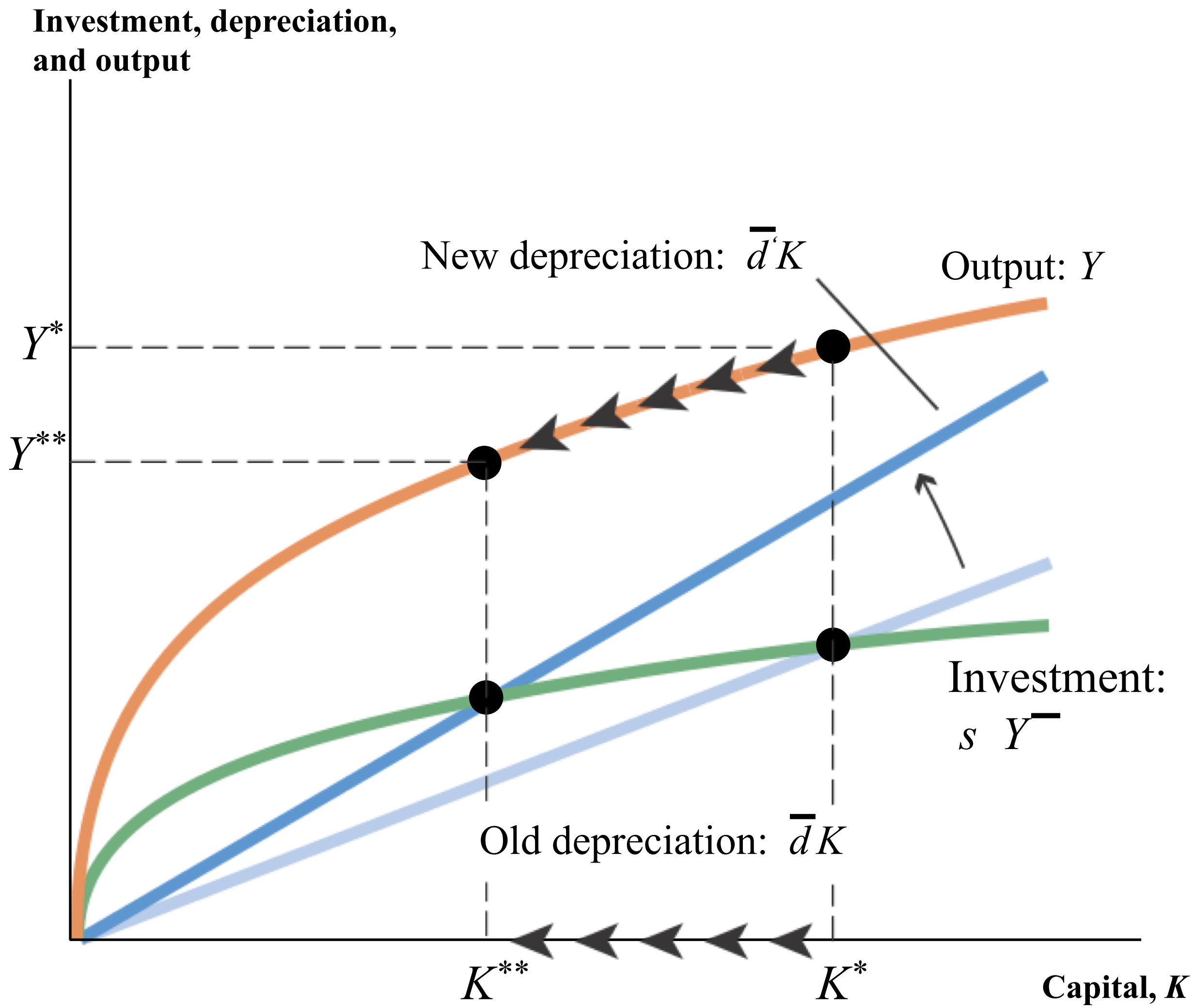
A Rise in the Depreciation Rate

- the depreciation rate is exogenously shocked to a higher rate
- the depreciation curve rotates upward and the investment curve remains unchanged
- the new steady state is located to the left: this means that depreciation exceeds investment
- the capital stock declines by transition dynamics until it reaches the new steady state
  - note that output declines rapidly at first but less rapidly as it converges to the new steady state

A Rise in the Depreciation Rate



The Behavior of Output Following an Increase in $\bar{d}$



(a) The Solow diagram with output.

Strengths and Weaknesses of the Solow Model

- The strengths of the Solow model are:
  1. It provides a theory that determines how rich a country is in the long run.
  2. The principle of transition dynamics allows for an understanding of differences in growth rates across countries.
- The weaknesses of the Solow model are:
  1. It focuses on investment and capital, while the much more important factor of TFP is still unexplained.
  2. It does not explain why different countries have different investment and productivity rates.
  3. The model does not provide a theory of sustained long-run economic growth.

NOW YOU TRY

Approaching k^* from above

Draw the Solow model diagram,
labeling the steady state $\boldsymbol{k^*}$.

On the horizontal axis, pick a value greater than $\boldsymbol{k^*}$ for the economy's initial capital stock. Label it $\boldsymbol{k_1}$.

Show what happens to $\boldsymbol{k}$ over time.
Does $\boldsymbol{k}$ move toward the steady state or away from it? (3 minutes to do this; and one comes forward to show our class.)

A numerical example

Production function (aggregate):

$$Y = F(K, L) = \sqrt{K \times L} = K^{1/2} L^{1/2}$$

To derive the per-worker production function, divide through by L:

Then substitute $y = Y/L$ and $k = K/L$ to get

A numerical example, *cont.*

Assume:

- ***s*** = 0.3
- δ = 0.1
- initial value of ***k*** = 4.0

Approaching the steady state:

A numerical example

Year	*k*	*y*	*c*	*i*	⍰*k*	Δ*k*
1	4.000	2.000	1.400	______	______	______
2	4.200	2.049	1.435	______	______	______
3	4.395	2.096	1.467	0.629	0.440	0.189
4	4.584	2.141	1.499	______	______	______
…						
10	5.602	2.367	1.657	0.710	0.560	0.150
…						
25	7.351	2.706	1.894	0.812	0.732	0.080
…						
100	8.962	2.994	2.096	0.898	0.896	0.002
…						
⍰	9.000	3.000	2.100	0.900	0.900	0.000

NOW YOU TRY

Solve for the steady state

Continue to assume

$s = 0.3$, $\delta = 0.1$, and $y = k^{1/2}$

Use the equation of motion

$$\Delta k = s\, f(k) - \delta k$$

to solve for the steady-state values of ***k***, ***y***, and ***c***.

Solve for the steady state

$\Delta \boldsymbol{k} = 0$ def. of steady state

An increase in the saving rate

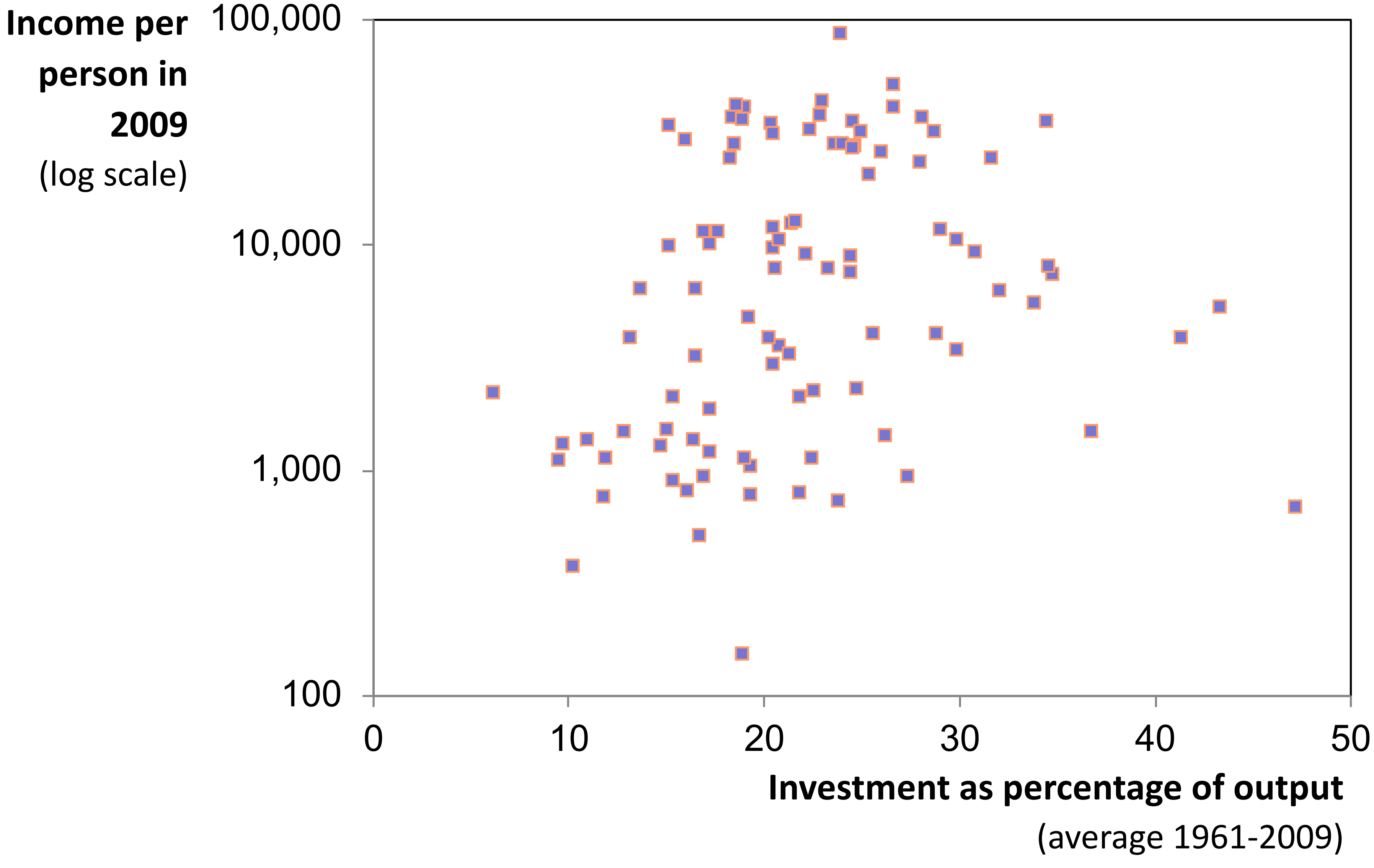
An increase in the saving rate raises investment…

…causing ***k*** to grow toward a new steady state:

Prediction:

- Higher $\boldsymbol{s}$ $\Rightarrow$ ___________
- And since $\boldsymbol{y} = \boldsymbol{f(k)}$,
  higher $\boldsymbol{k}^*$ $\Rightarrow$ ___________ .
- Thus, the Solow model predicts that countries with
  _______________ and ___________
  will have higher levels of capital and income per worker in the long run.

International evidence on investment rates and income per person



The Golden Rule: Introduction

- Different values of ***s*** lead to different steady states. How do we know which is the "best" steady state?

  (Class discussion)

- The "best" steady state has ….

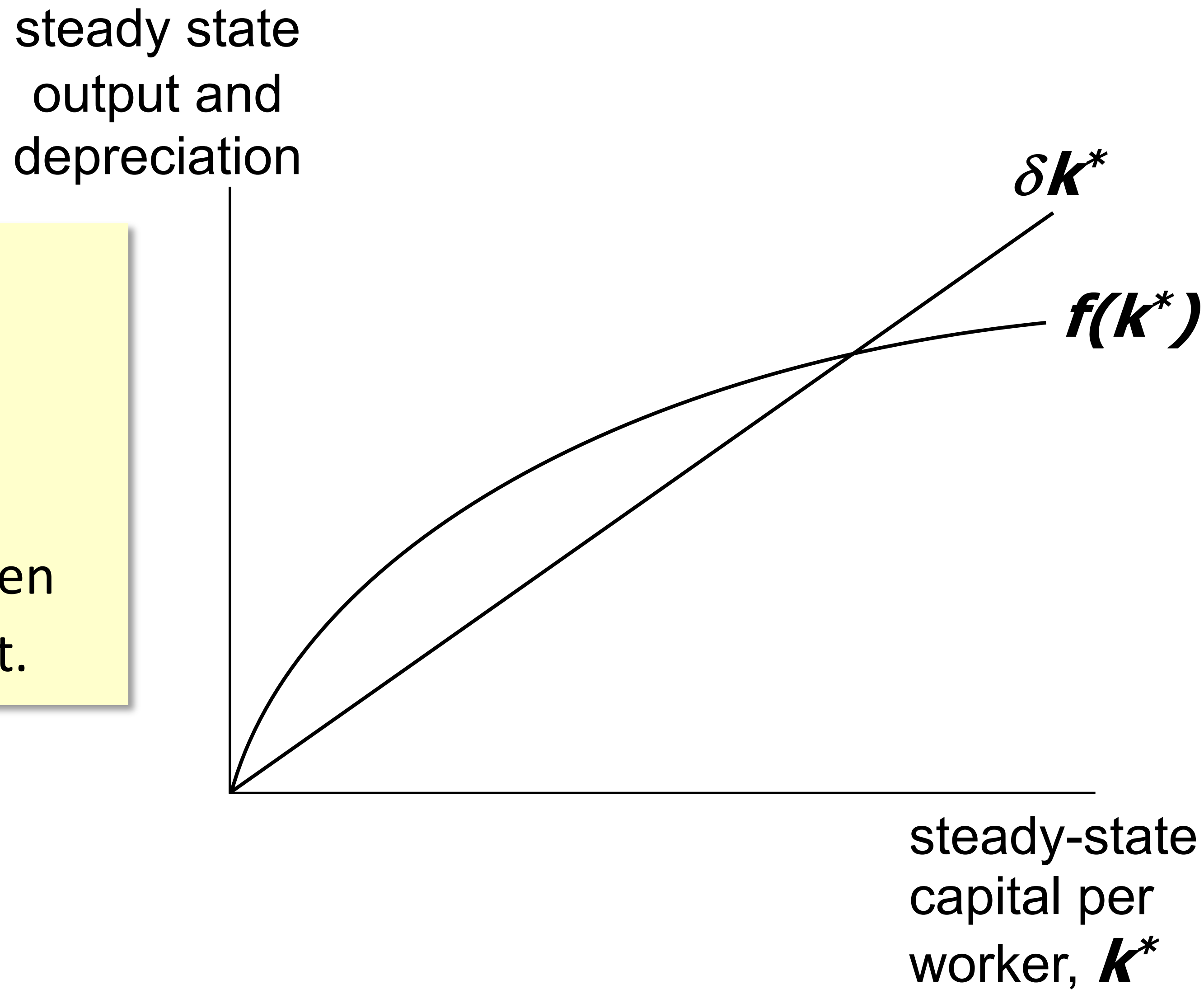
- An increase in ***s***
  - leads to higher ***k**** and ***y****, which __________ ***c****
  - reduces consumption's share of income (1–***s***), which __________ ***c*.***
- So, how do we find the ***s*** and ***k**** that maximize ***c****?

The Golden Rule capital stock

k^*_{gold} = the **Golden Rule level of capital**,
the steady state value of k
that maximizes consumption.

To find it, first express c^* in terms of k^*:

The Golden Rule capital stock



Then, graph $f(k^*)$ and δk^*, look for the point where the gap between them is biggest.

The Golden Rule capital stock

$c^* = f(k^*) - \delta k^*$

is biggest where the slope of the production function

equals

the slope of the depreciation line:

____

steady-state capital per worker, k^*

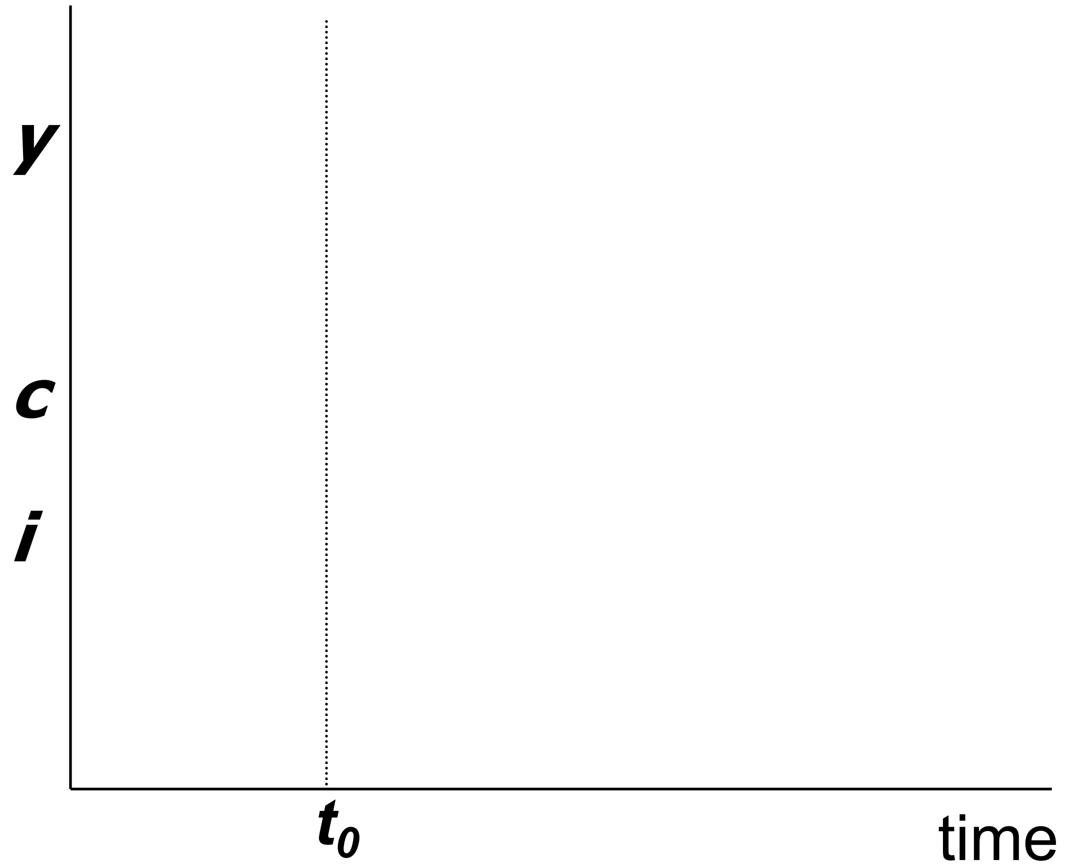
The transition to the Golden Rule steady state

- The economy does NOT have a tendency to move toward the Golden Rule steady state.
- Achieving the Golden Rule requires that policymakers adjust _____.
- This adjustment leads to a new steady state with _________ consumption.
- But what happens to consumption during the transition to the Golden Rule?

Starting with too much capital

If $\boldsymbol{k}^* > \boldsymbol{k}^*_{gold}$ then increasing $\boldsymbol{c}^*$ requires a fall in ___.

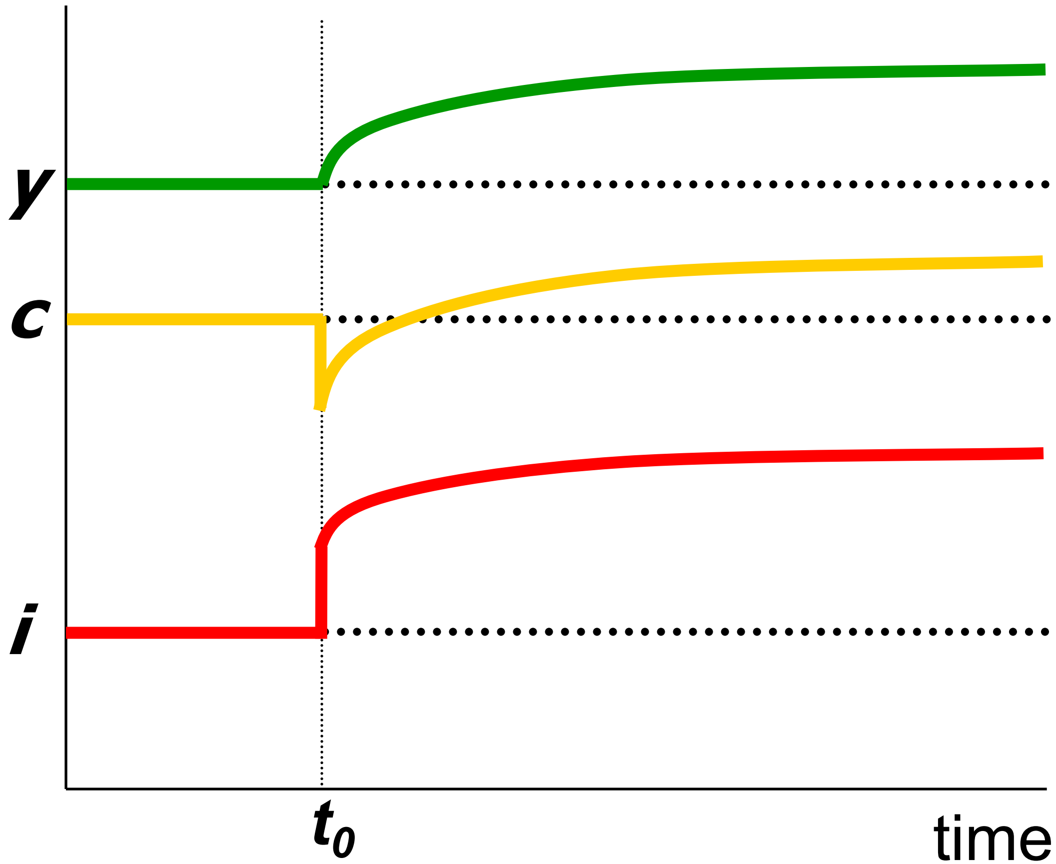
In the transition to the Golden Rule, consumption is higher at all points in time.



Starting with too little capital

If $\boldsymbol{k}^* < \boldsymbol{k}^*_{gold}$
then increasing $\boldsymbol{c}^*$ requires an increase in $\boldsymbol{s}$.

Future generations enjoy __________ consumption, but the current one experiences an initial _________ in consumption.



Population growth

- Assume the population and labor force grow at rate ***n*** (exogenous):

- EX: Suppose ***L*** = 1,000 in year 1 and the population is growing at 2% per year (***n*** = 0.02).
- Then Δ***L*=**,
so ***L*** = __________ in year 2.

Break-even investment

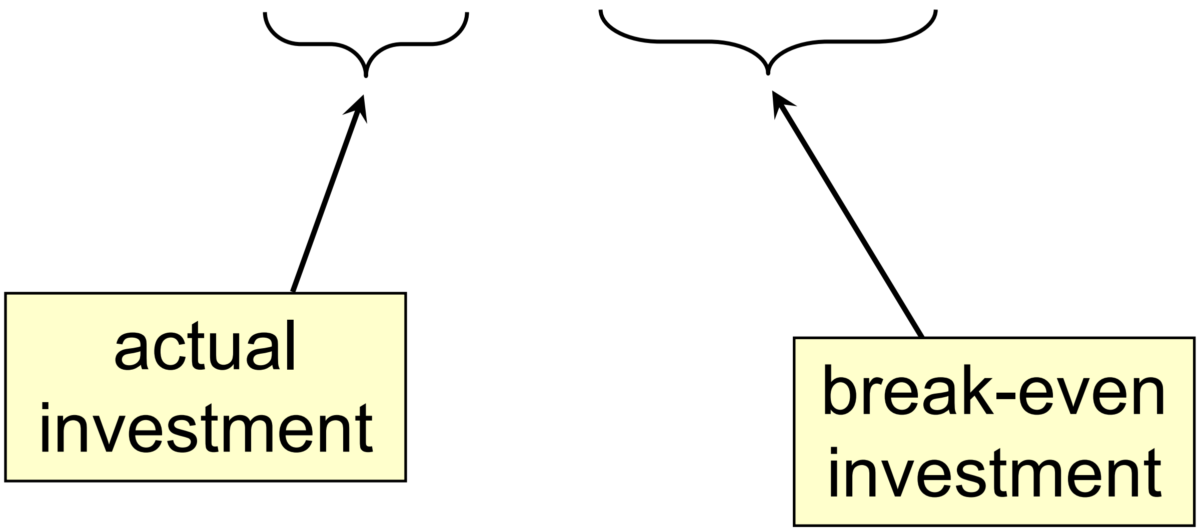
- ____________= **break-even investment**, the amount of investment necessary to keep ***k*** constant.
- Break-even investment includes:
  - δ***k*** to replace capital as it wears out
  - ***nk*** to equip new workers with capital

  (Otherwise, ***k*** would fall as the existing capital stock is spread more thinly over a larger population of workers.)

The equation of motion for *k*

- With population growth,
  the equation of motion for ***k*** is:

$$\Delta \boldsymbol{k} = \underline{\hspace{8cm}}$$



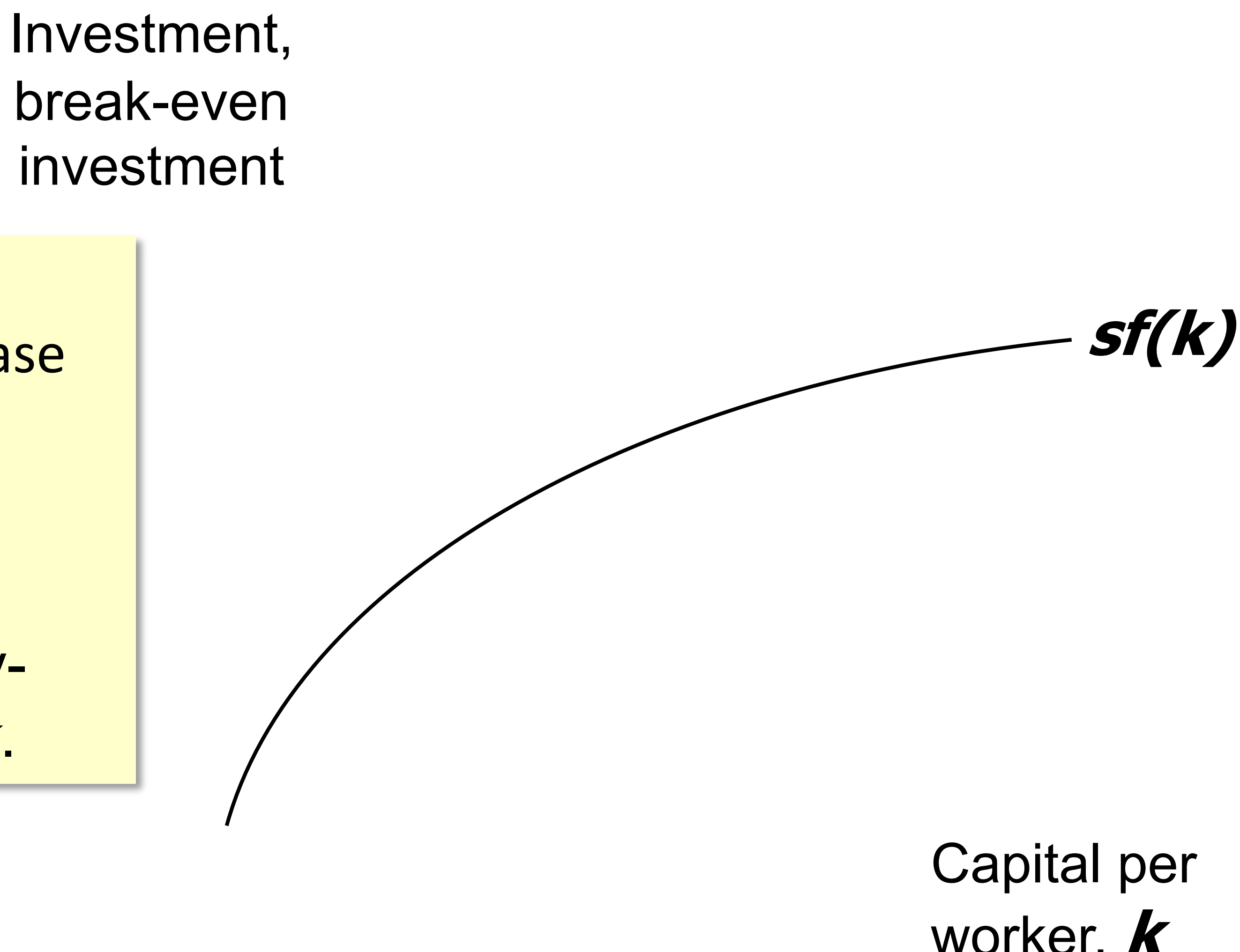
The Solow model diagram

$$\Delta \boldsymbol{k} = \boldsymbol{s}\, \boldsymbol{f(k)} - (\delta + \boldsymbol{n})\boldsymbol{k}$$

The impact of population growth

An increase in ***n*** causes an increase in break-even investment,

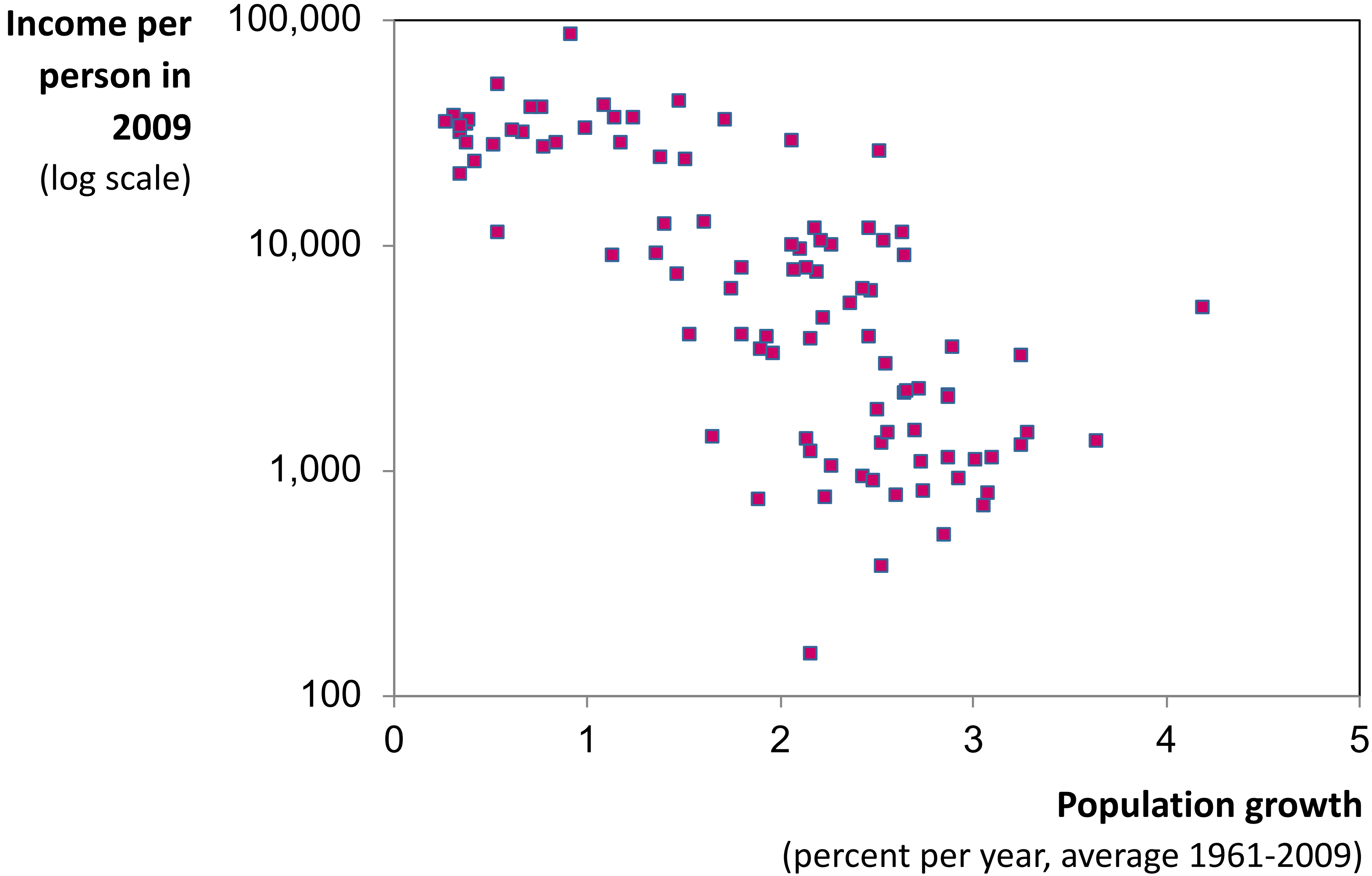
leading to a _______ steady-state level of ***k***.



Prediction:

- Higher ***n*** $\Rightarrow$ ___________.
- And since ***y*** = f(***k***) ,
  lower ***k**** $\Rightarrow$ __________.
- Thus, the Solow model predicts that countries with higher population growth rates will have _______ levels of capital and income per worker in the long run.

International evidence on population growth and income per person



The Golden Rule with population growth

To find the Golden Rule capital stock, express ***c**** in terms of ***k****:

c* is maximized when

or equivalently,

In the Golden Rule steady state, the marginal product of capital net of depreciation equals the population growth rate.

Alternative perspectives on population growth

The Malthusian Model (1798)

- Predicts population growth will outstrip the Earth's ability to produce food, leading to the impoverishment of humanity.
- Since Malthus, world population has increased sixfold, yet living standards are _____ than ever.
- Malthus neglected the effects of ___________ _________

Alternative perspectives on population growth

The Kremerian Model (1993)

- Posits that population growth contributes to economic growth.
- More people = more geniuses, scientists & engineers, so faster ___________ _________.
- Evidence, from very long historical periods:
  - As world pop. growth rate increased, so did rate of growth in living standards
  - Historically, regions with larger populations have enjoyed faster growth.