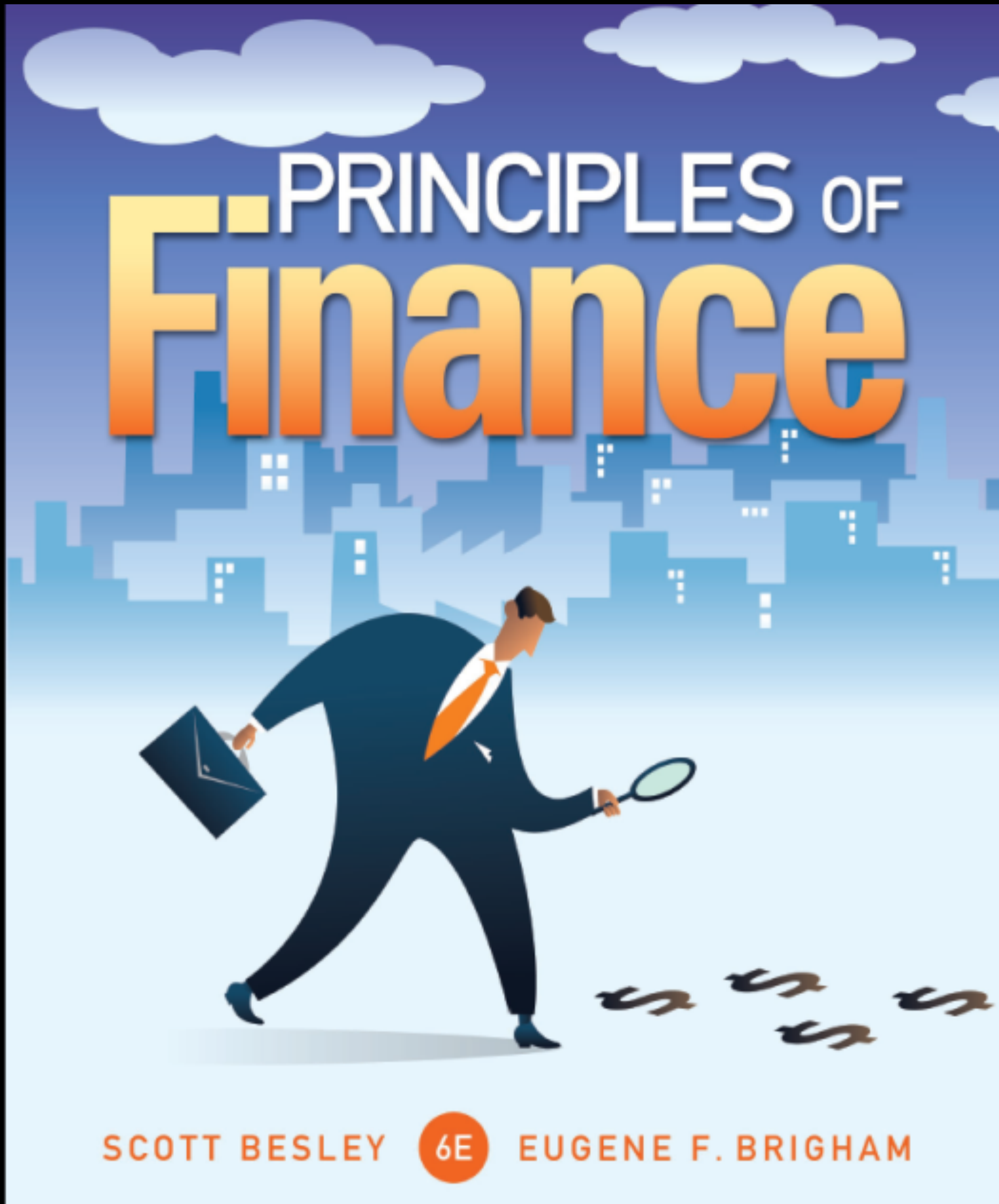


Chapter 9

The Time Value of Money

Besley & Brigham

Chapter 9- Learning Objectives

- ✓ Identify various types of cash flow patterns (streams) that are observed in business.
- ✓ Compute (a) the future values and (b) the present values of different cash flow streams and explain the results.
- ✓ Compute (a) the return (interest rate) on an investment (loan) and (b) how long it takes to reach a financial goal.
- ✓ Explain the difference between the Annual Percentage Rate (APR) and the Effective Annual Rate (EAR) and explain when each is more appropriate to use.
- ✓ Describe an amortized loan and compute (a) amortized loan payments and (b) the balance (amount owed) on an amortized loan at a specific point during its life.

Time Value of Money

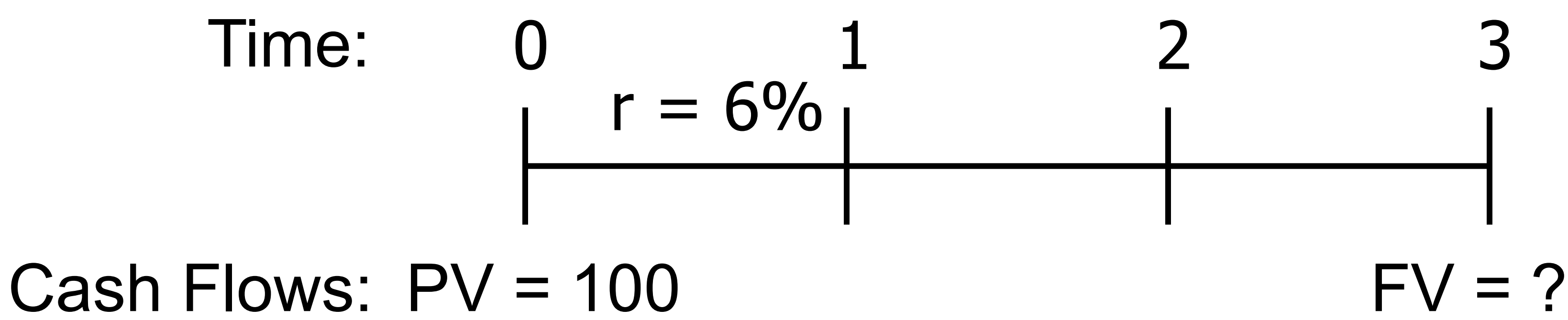
- The principles and computations used to revalue cash payoffs at different times so they are stated in dollars of the same time period
- The most important concept in finance used in nearly every financial decision
  - Business decisions
  - Personal finance decisions

Cash Flow Patterns

- ✓ Lump-sum amount – a single payment paid or received in the current period or some future period
- ✓ Annuity - A series of equal payments that occur at equal time intervals
- ✓ Uneven cash flow stream – multiple payments that are not equal, do not occur at equal intervals, or both conditions exist

Cash Flow Timelines

Graphical representations used to show timing of cash flows:



Time 0 is today, Time 1 is the end of Period 1 (beginning of Period 2), and so forth.

Future Value

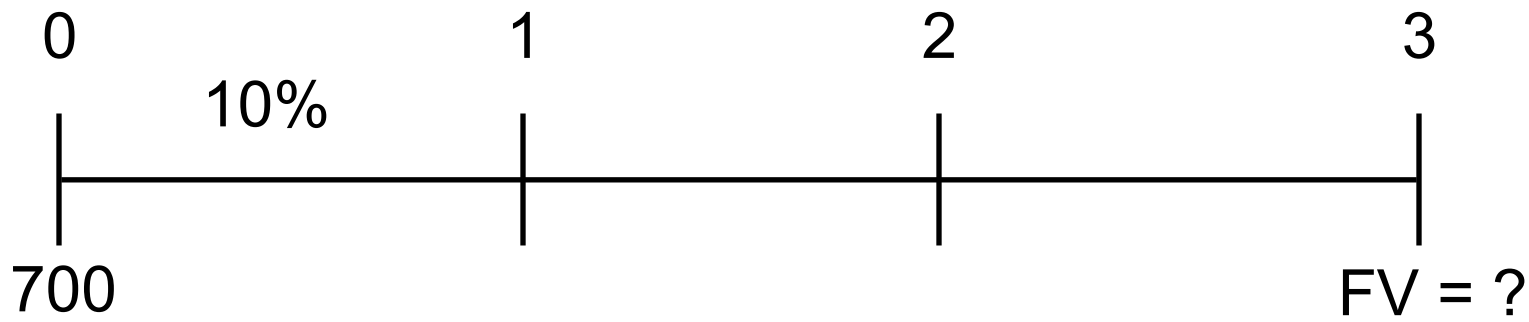
The amount to which a cash flow or series of cash flows will grow over a period of time when compounded at a given interest rate.

Future Value

- ✓ How much would you have at the end of one year if you deposit \$700 in a bank account that pays 10 percent interest each year?

$$FV_n = FV_1 = PV + INT$$
$$= PV + PV(r)$$
$$= PV(1 + r)$$
$$= \$700(1 + 0.10) = \$700(1.10) = \$770$$

What's the FV of an initial $700 after three years if r = 10%?



Finding FV is Compounding

Future Value

After 1 year:

$$FV_1 = PV + Interest_1 = PV + PV(r)$$
$$= PV(1 + r)$$
$$= \$700(1.10)$$
$$= \$770.00$$

After 2 years:

$$FV_2 = FV_1(1 + r)$$
$$= [PV(1 + r)](1 + r)$$
$$= PV(1 + r)^2$$
$$= \$700(1.10)^2 = \$700(1.2100)$$
$$= \$847.00$$

Future Value

After 3 years:

$$
\begin{aligned}
FV_3 &= FV_2(1 + r) \\
&= [PV(1 + r)^2](1 + r) \\
&= PV(1 + r)^3 \\
&= \$700(1.10)^3 = \$700(1.331) \\
&= \$931.70
\end{aligned}
$$

In general, $FV_n = PV(1 + r)^n$

Three Ways to Solve Time Value of Money Problems

- ✓ Use Equations
- ✓ Use Financial Calculator
- ✓ Use Electronic Spreadsheet

Numerical (Equation) Solution

$$FV_n = PV(1+r)^n$$

PV = \$700, r = 10%, and n =3

$$FV_n = \$700(1.10)^3$$

$$= \$700(1.3310) = \$931.70$$

Financial Calculator Solution

The equation $FV_n = PV(1 + r)^n$ is programmed into the calculator; you must provide the numbers for the calculator to perform the computation

Financial Calculator Solution

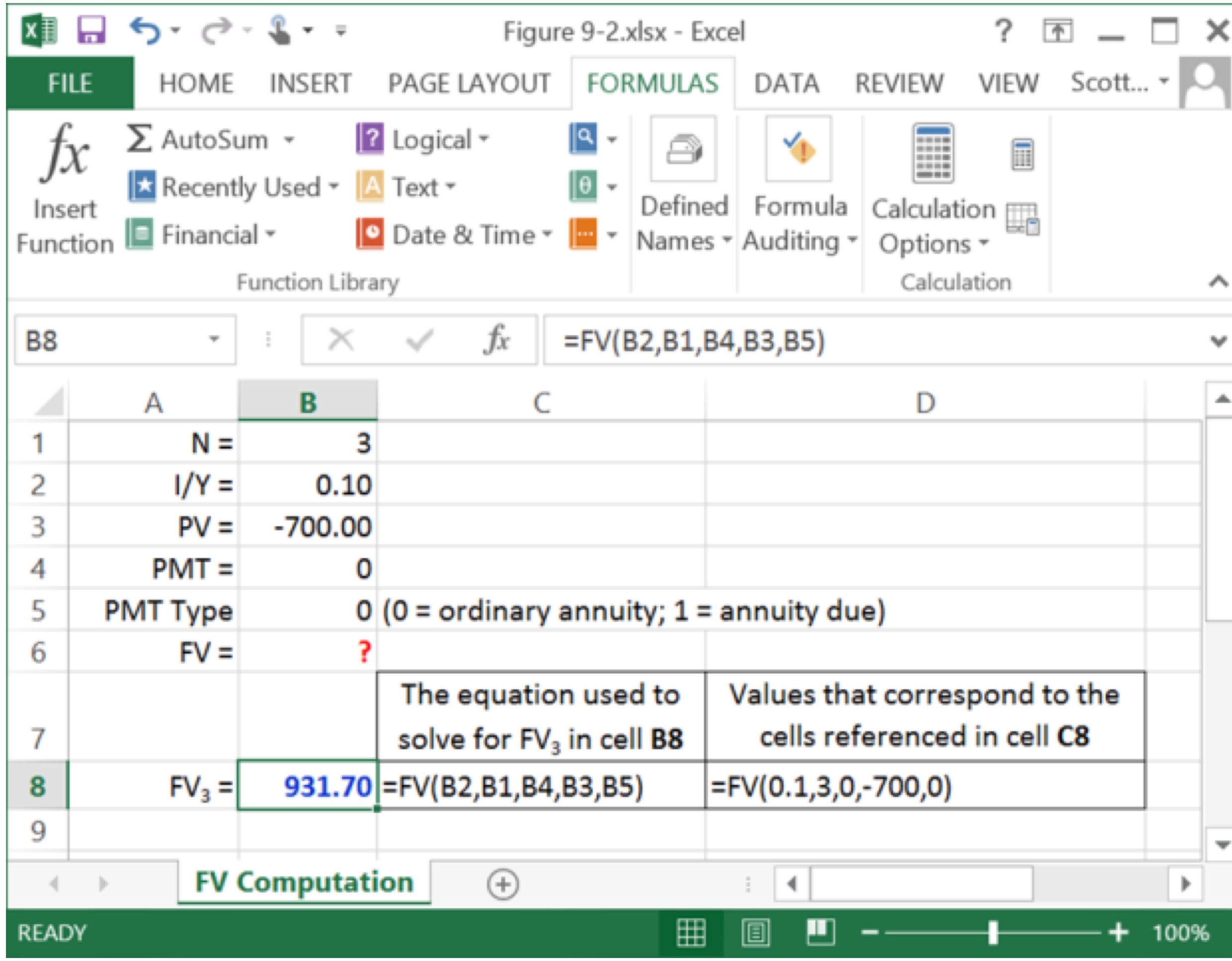
Here's the setup to find FV:

INPUTS	3	10	-700	0	?
	N	I/Y	PV	PMT	FV
OUTPUT					931.70

Clearing automatically sets everything to 0, but for safety enter PMT = 0.

Spreadsheet Solution

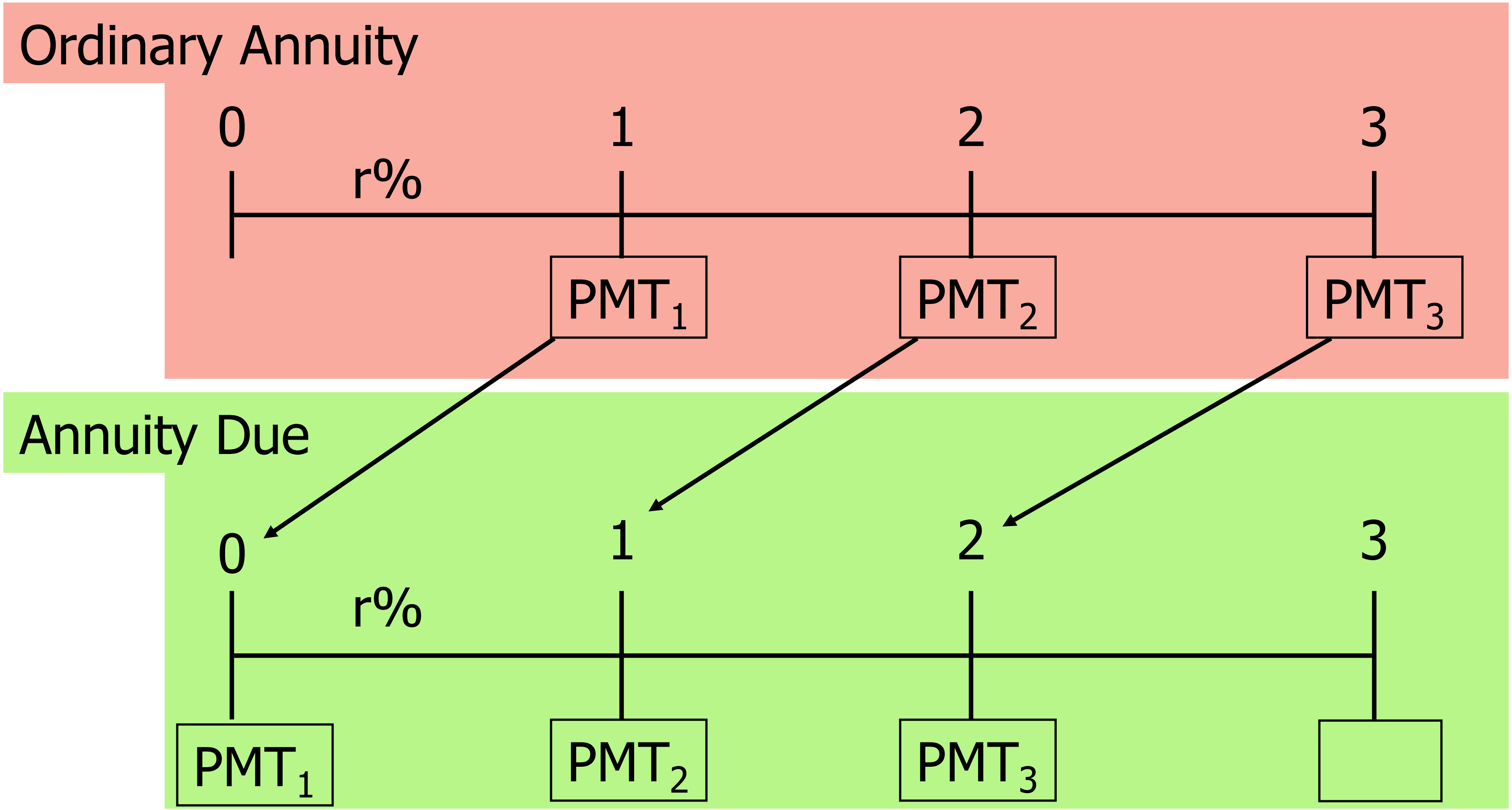
The input values must be entered in a specific order: I/Y, N, PMT, PV, and PMT type (not used for this problem).



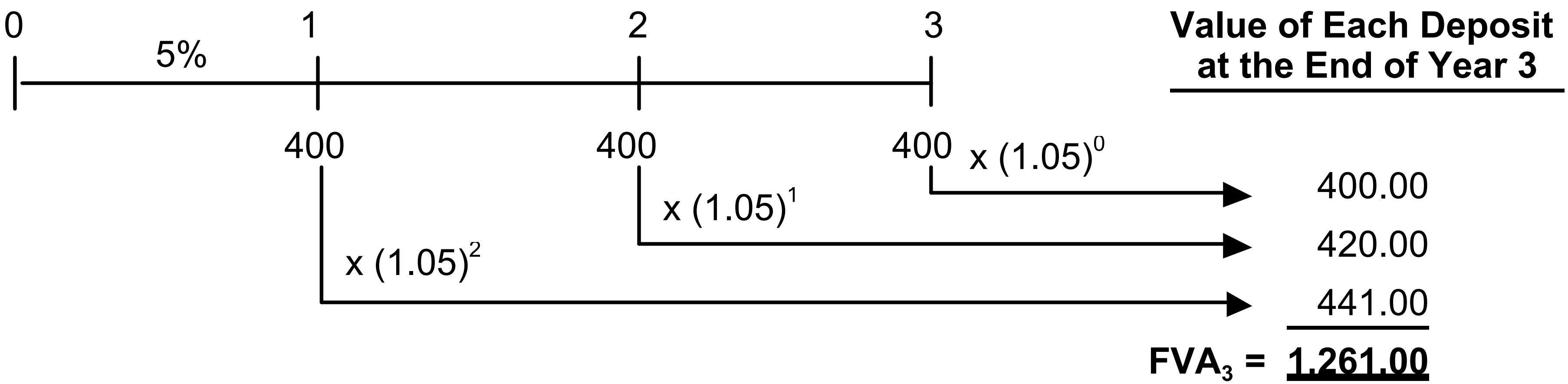
Future Value of an Annuity

- ✓ **Annuity:** A series of payments of equal amounts at equal intervals for a specified number of periods.
- ✓ **Ordinary (deferred) Annuity:** An annuity whose payments occur at the *end* of each period.
- ✓ **Annuity Due:** An annuity whose payments occur at the *beginning* of each period.

Ordinary Annuity versus Annuity Due



FV of a 3-year Ordinary Annuity of $400 at 5%



Numerical Solution

$$FVA_n = PMT\left[\sum_{t=0}^{n-1}(1+r)^t\right] = PMT\left[\frac{(1+r)^n - 1}{r}\right]$$

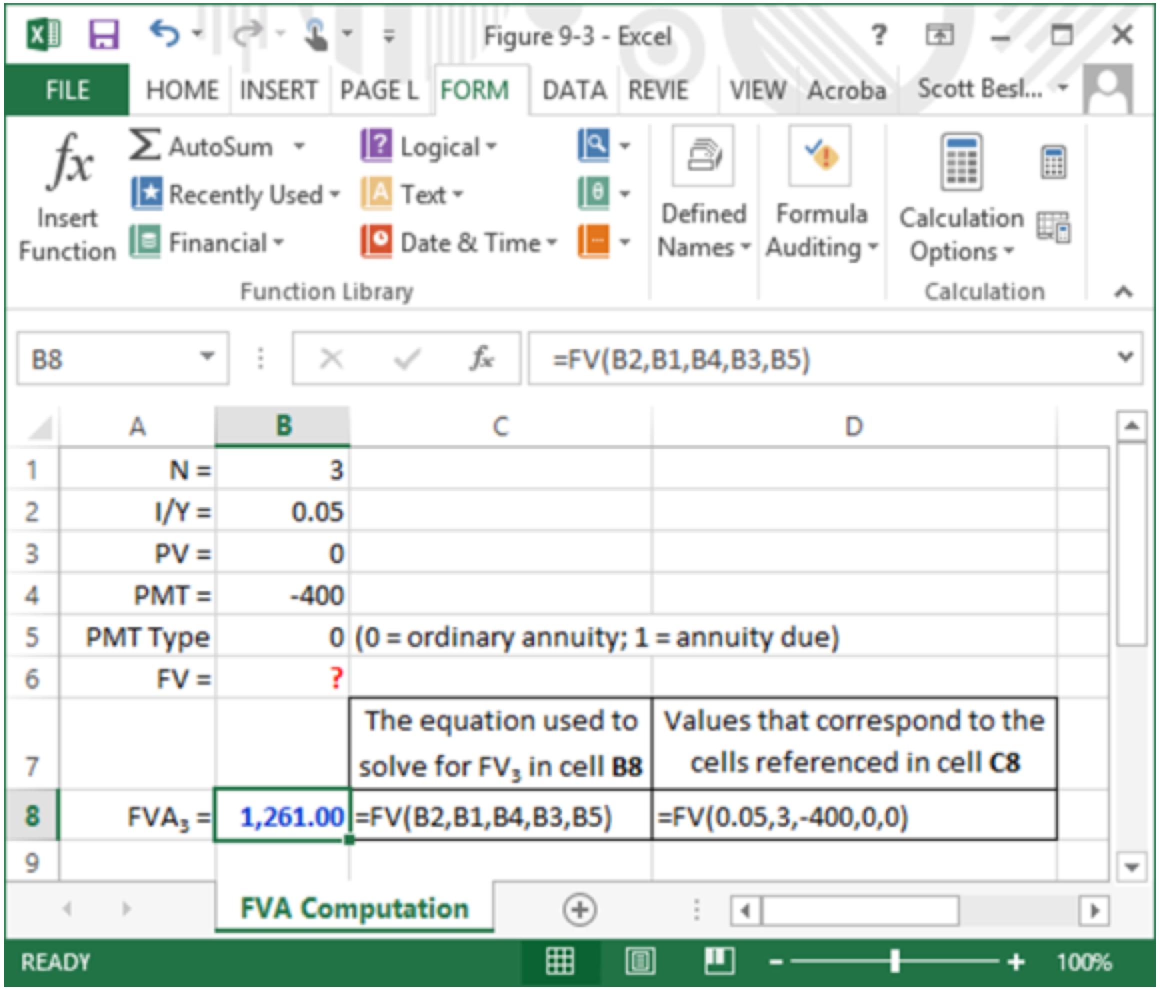
$$FVA_3 = \$400\left[\frac{(1.05)^3 - 1}{0.05}\right]$$

$$= \$400(3.1525) = \$1261.00$$

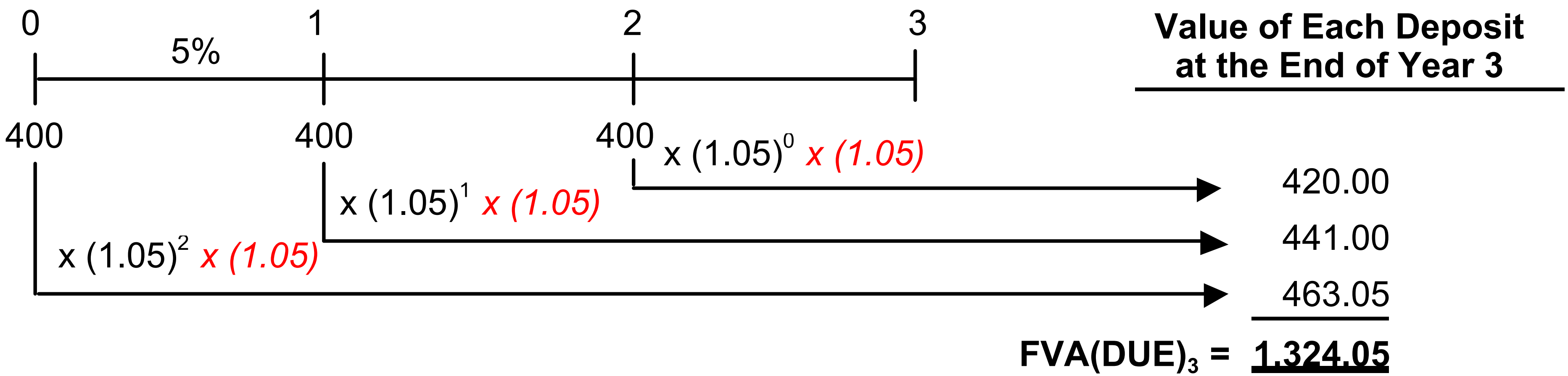
Financial Calculator Solution

	N	I/Y	PV	PMT	FV
INPUTS	3	5	0	-400	?
OUTPUT					1,261.00

Spreadsheet Solution



FV of a 3-year Annuity Due of $400 at 5%



Numerical Solution—FVA(DUE)

$$FVA(DUE)_n = PMT\left\{\left[\frac{(1+r)^n - 1}{r}\right] \times (1+r)\right\}$$

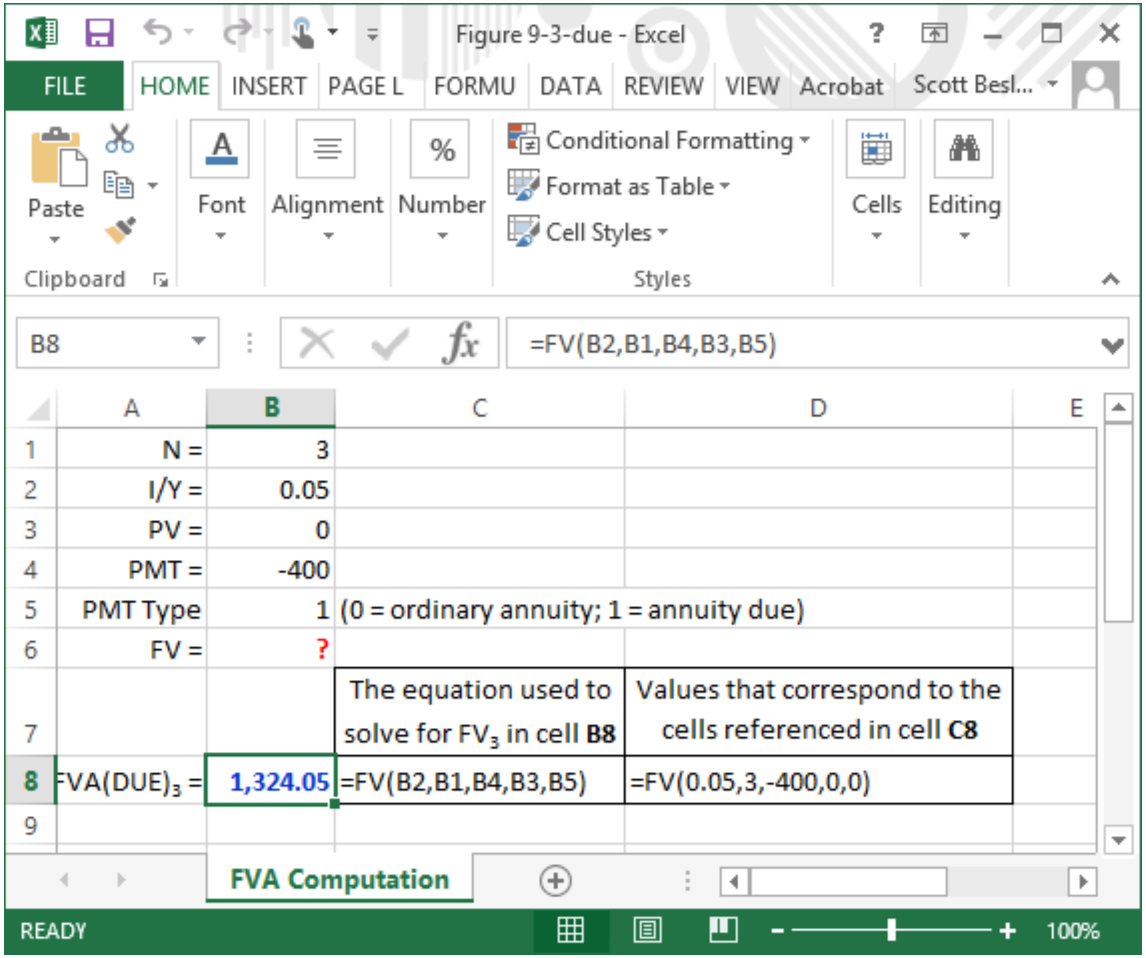
$$FVA(DUE)_3 = \$400\left\{\left[\frac{(1.05)^3 - 1}{0.05}\right] \times (1.05)\right\}$$

$$= \$400(3.310125) = \$1{,}324.05$$

Financial Calculator Solution—FVA(DUE)

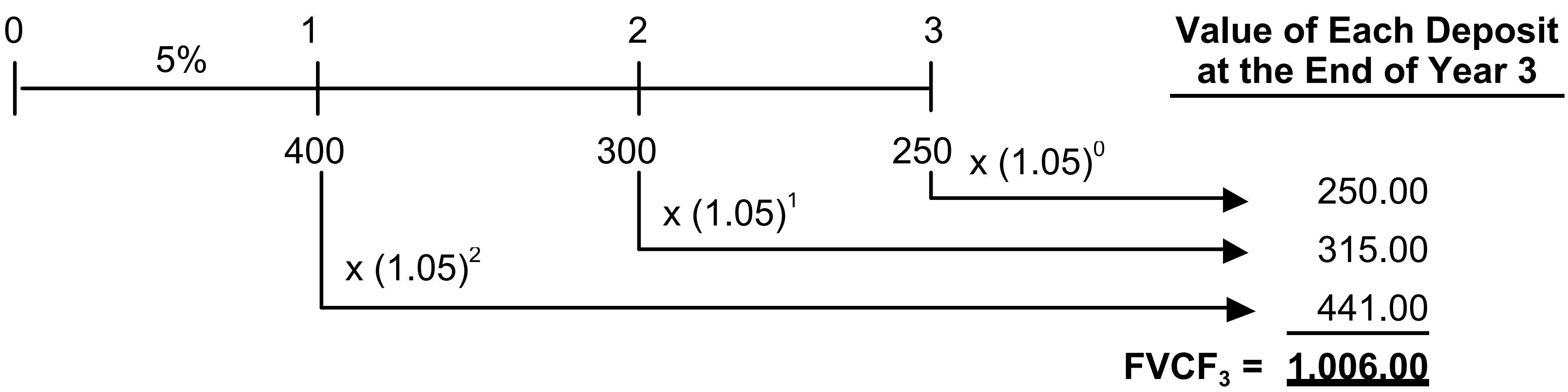
					BGN
INPUTS	3	5	0	-400	?
	N	I/Y	PV	PMT	FV
OUTPUT					1,324.05

Spreadsheet Solution—FVA(DUE)



	A	B	C	D
1	N =	3		
2	I/Y =	0.05		
3	PV =	0		
4	PMT =	-400		
5	PMT Type	1	(0 = ordinary annuity; 1 = annuity due)	
6	FV =	?		
7			The equation used to solve for FV_3 in cell **B8**	Values that correspond to the cells referenced in cell **C8**
8	$FVA(DUE)_3$ =	1,324.05	=FV(B2,B1,B4,B3,B5)	=FV(0.05,3,-400,0,0)
9				

Future Value of an Uneven Cash Flow



Numerical Solution

$$FVCF_n = CF_1\left(1+r\right)^{n-1} + CF_2\left(1+r\right)^{n-2} + L + CF_n\left(1+r\right)^0 = \sum_{t=1}^{n} CF_t\,(1+r)^{n-t}$$

Solving for interest r and n

- ✓ The variables in the equations are labeled PV, FV, r, n, and PMT
- ✓ If we know the values of all of these variables except one, we can solve for the unknown variable

If an investment of $100 grows to $165.50 in eight years, what rate of return is earned?

Solve for r in the following equation:

$$FV_n = PV(1 + r)^n$$

$$\$165.50 = \$100(1 + r)^8$$

Financial calculator solution:

INPUTS	**8**	**?**	**-100**	**0**	**165.50**
	N	**I/Y**	**PV**	**PMT**	**FV**
OUTPUT		**6.5**			

If sales grow at 12.25% per year, how long before sales double?

Solve for n:

$$FV_n = 1(1 + r)^n$$

$$2 = 1(1.1225)^n$$

Financial calculator solution:

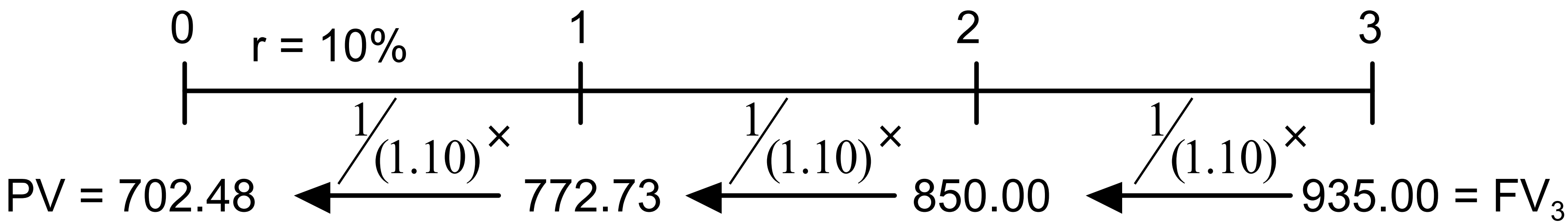
	N	I/Y	PV	PMT	FV
INPUTS	?	12.25	-1	0	2
OUTPUT	6.0				

Present Value

- **Present value** is the value today of a future cash flow or series of cash flows.

- **Discounting** is the process of finding the present value of a future cash flow or series of future cash flows; it is the reverse of compounding.

What is the PV of $935 due in three years if r = 10%?



Numerical Solution

Solve $FV_n = PV (1 + r)^n$ for PV:

$$PV = \frac{FV_n}{(1+r)^n} = FV_n\left(\frac{1}{1+r}\right)^n$$

$$PV = \$935\left(\frac{1}{1.10}\right)^3$$

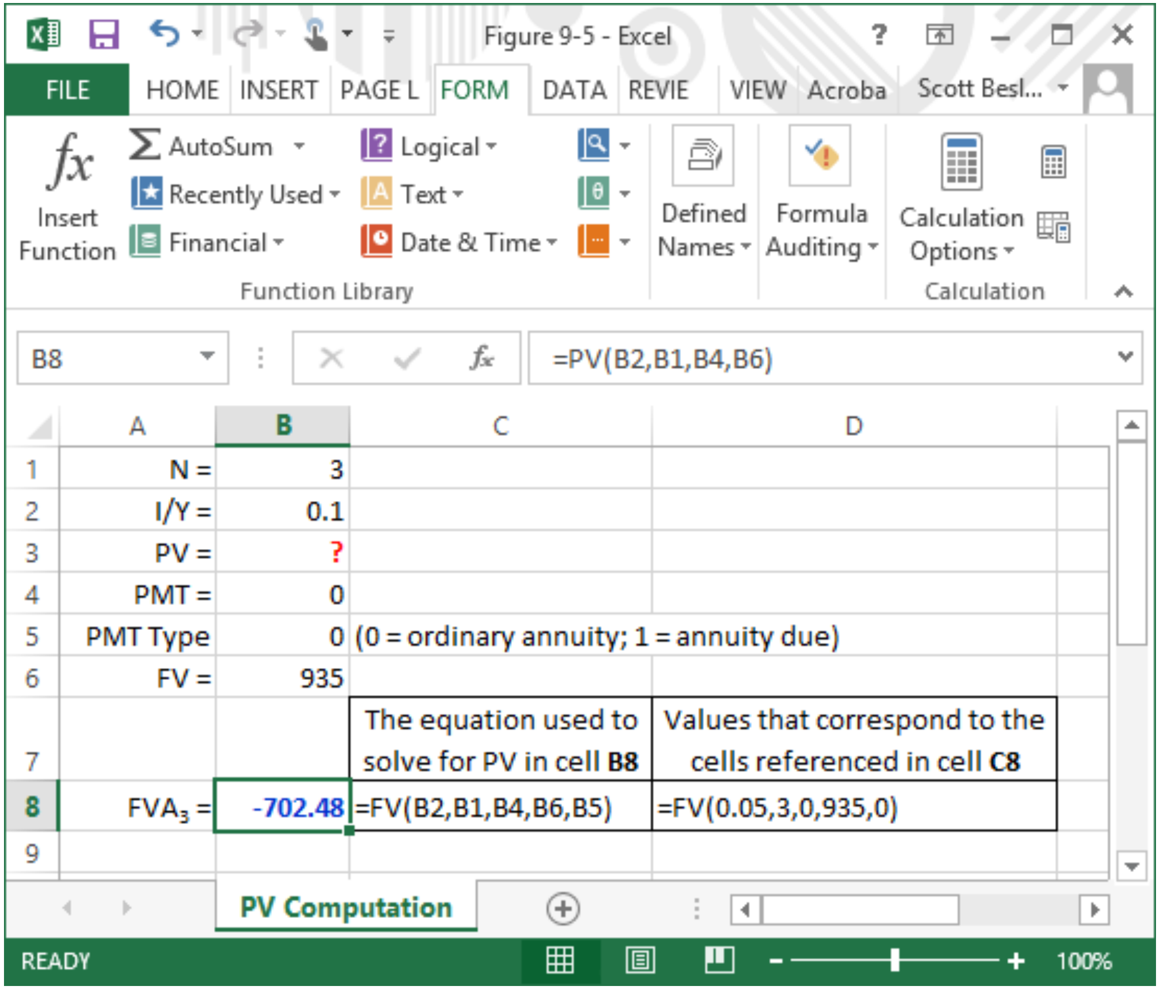
$$= \$935(0.7513) = \$702.48$$

Financial Calculator Solution

INPUTS	3	10	?	0	935
	N	I/Y	PV	PMT	FV
OUTPUT			-702.48		

Either PV or FV must be negative.
Here PV = -702.48; invest $702.48 today, take out $935 after 3 years.

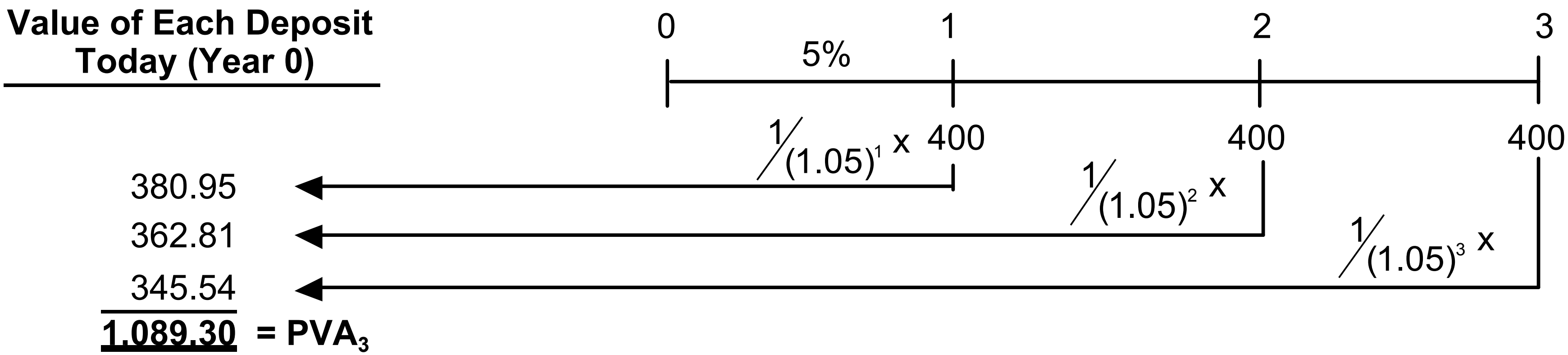
Spreadsheet Solution



Present Value of an Annuity

- PVA_n = the present value of an annuity with n payments
- Each payment is discounted, and the sum of the discounted payments is the present value of the annuity

PV of a 3-year Ordinary Annuity of $400 at 5%



Numerical Solution

$$PVA_n = PMT\left[\sum_{t=1}^{n} \frac{1}{(1+r)^t}\right] = PMT\left[\frac{1 - \frac{1}{(1+r)^n}}{r}\right]$$

$$PVA_3 = \$400\left[\frac{1 - \frac{1}{(1.05)^3}}{0.05}\right]$$

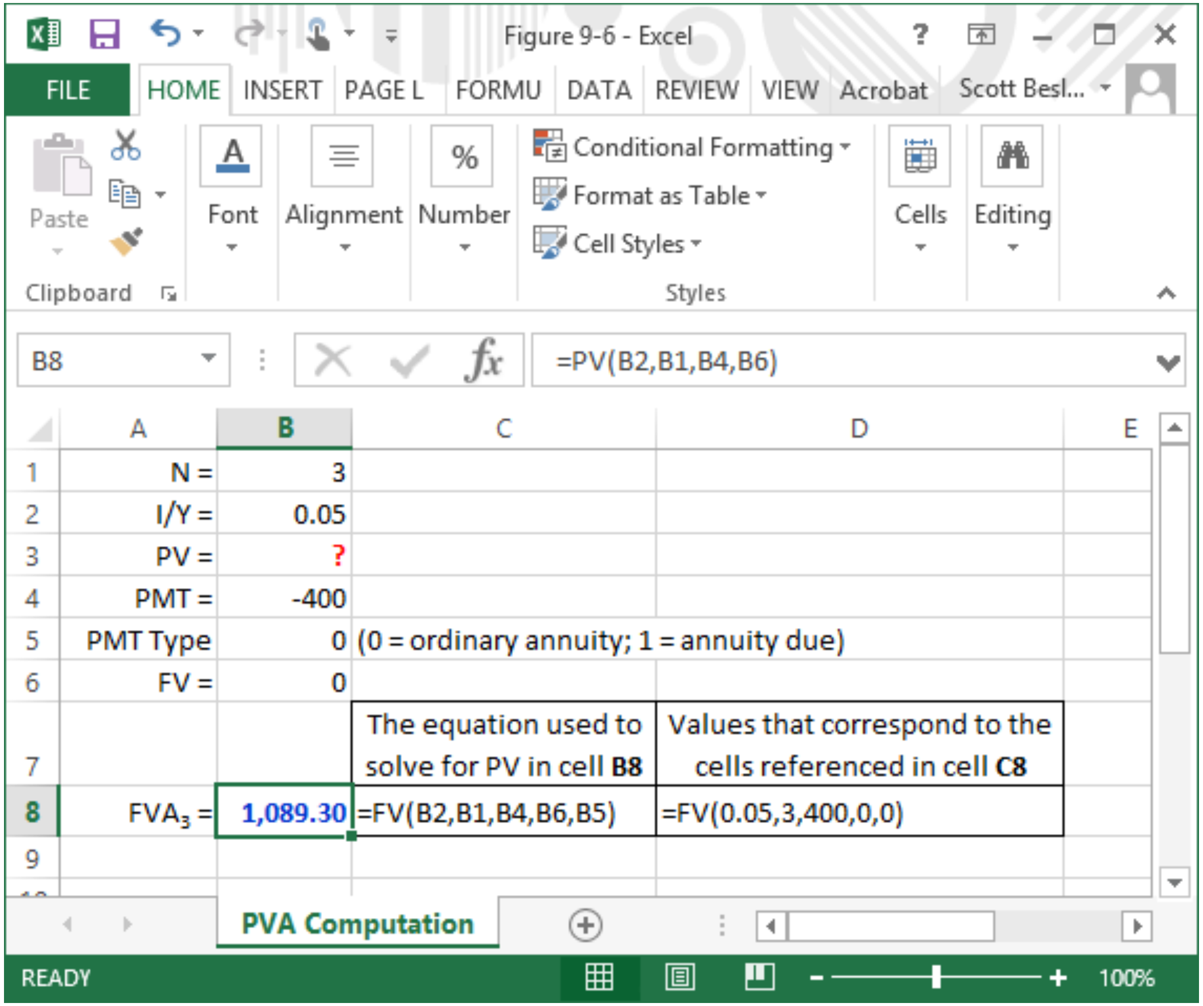
$$= \$400(2.72325) = \$1089.30$$

Financial Calculator Solution

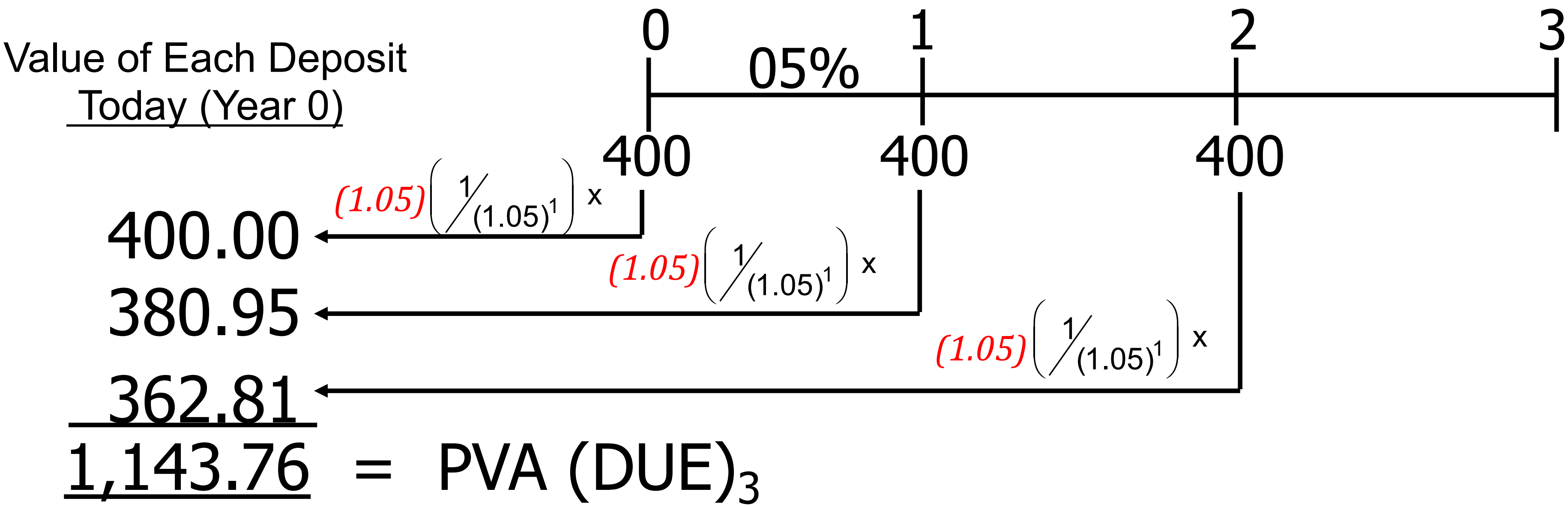
INPUTS	3	5	?	400	0
	N	I/Y	PV	PMT	FV
OUTPUT			-1,089.30		

We know the payments but there is no lump sum FV, so enter 0 for future value.

Spreadsheet Solution



Present Value of an Annuity Due



Numerical Solution—PVA(DUE)

$$PVA(DUE)_n = PMT\left\{\left[\frac{1-\frac{1}{(1+r)^n}}{r}\right]\times\boldsymbol{(1+r)}\right\}$$

$$PVA(DUE)_3 = 400\left\{\left[\frac{1-\frac{1}{(1.05)^3}}{0.05}\right]\times\boldsymbol{(1.05)}\right\} = 400(2.85941) = 1,143.76$$

Financial Calculator Solution—PVA(DUE)

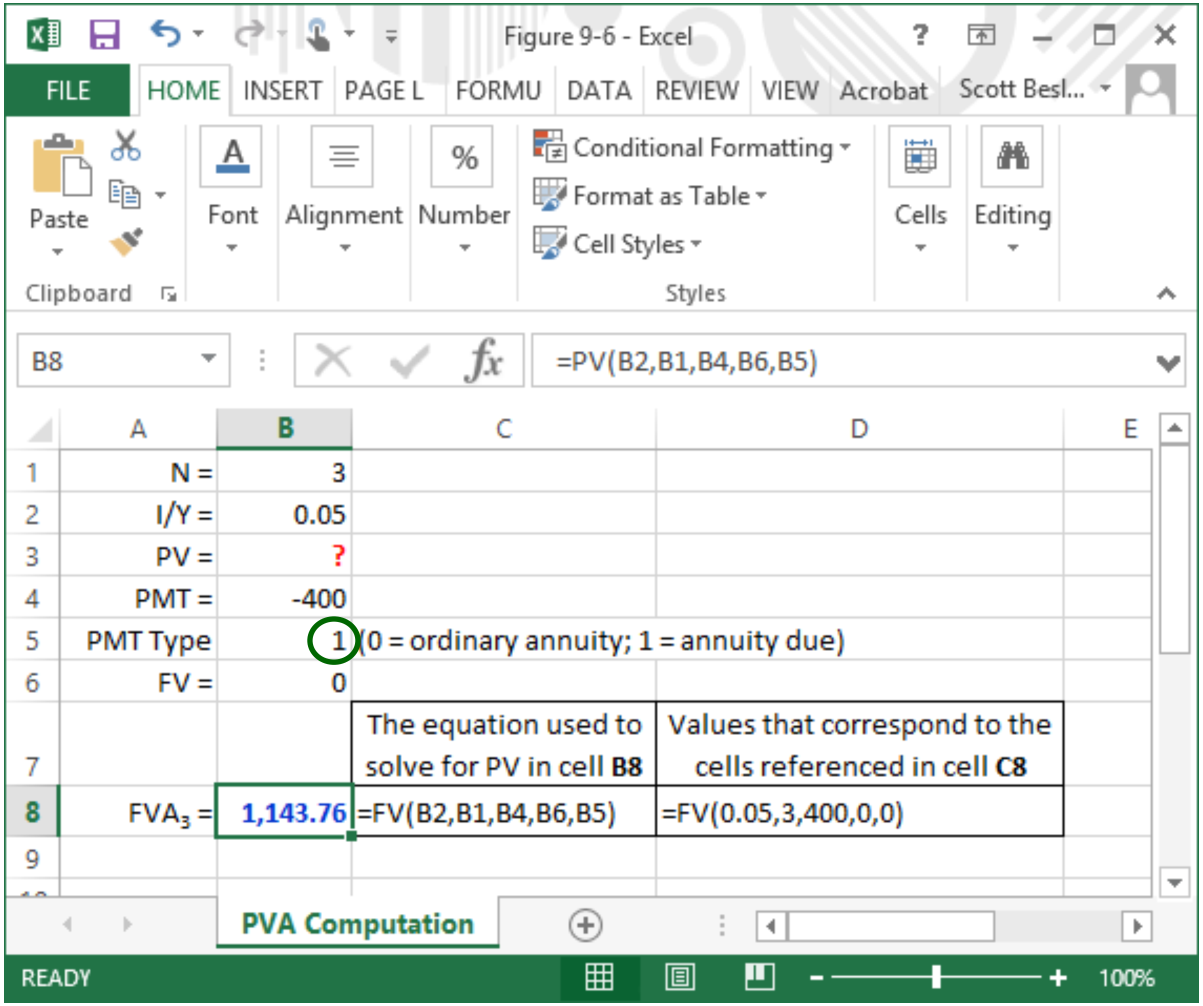
Switch from “End” to “Begin”.

Then enter variables to find PVA_3 = $1,143.76

					BGN
INPUTS	3	5	?	-400	0
	N	I/Y	PV	PMT	FV
OUTPUT			1,143.76		

Spreadsheet Solution—PVA(DUE)

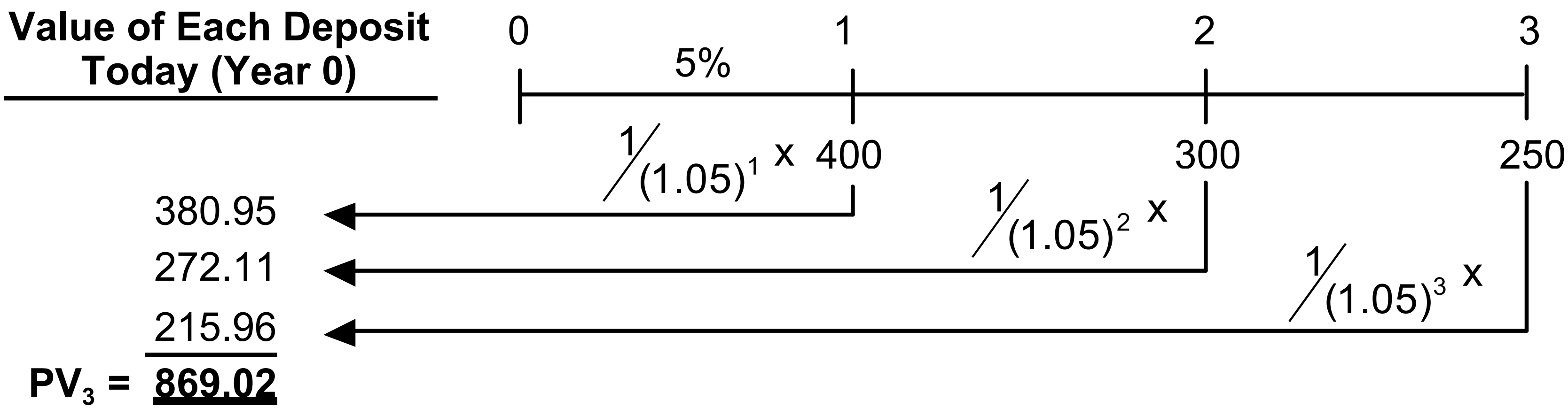
Insert a "1" for Type



Uneven Cash Flow Streams

- ✓ A series of cash flows in which the amount varies from one period to the next:
  - ✓ Payment (PMT) designates constant cash flows—that is, an annuity stream.
  - ✓ Cash flow (CF) designates cash flows in general, both constant cash flows and uneven cash flows.

Present Value of Uneven Cash Flow Stream



Numerical Solution

$$PVCF_n = CF_1\left[\frac{1}{(1+r)^1}\right] + CF_2\left[\frac{1}{(1+r)^2}\right] + \cdots + CF_n\left[\frac{1}{(1+r)^n}\right] = \sum_{t=1}^{n} CF_t\left[\frac{1}{(1+r)^t}\right]$$

$$PVCF_n = \frac{400}{(1.05)^1} + \frac{300}{(1.05)^2} + \frac{250}{(1.05)^3} = 400(0.95238) + 300(0.90703) + 250(0.86384)$$

$$= 380.952 + 272.109 + 215.960 = 869.02$$

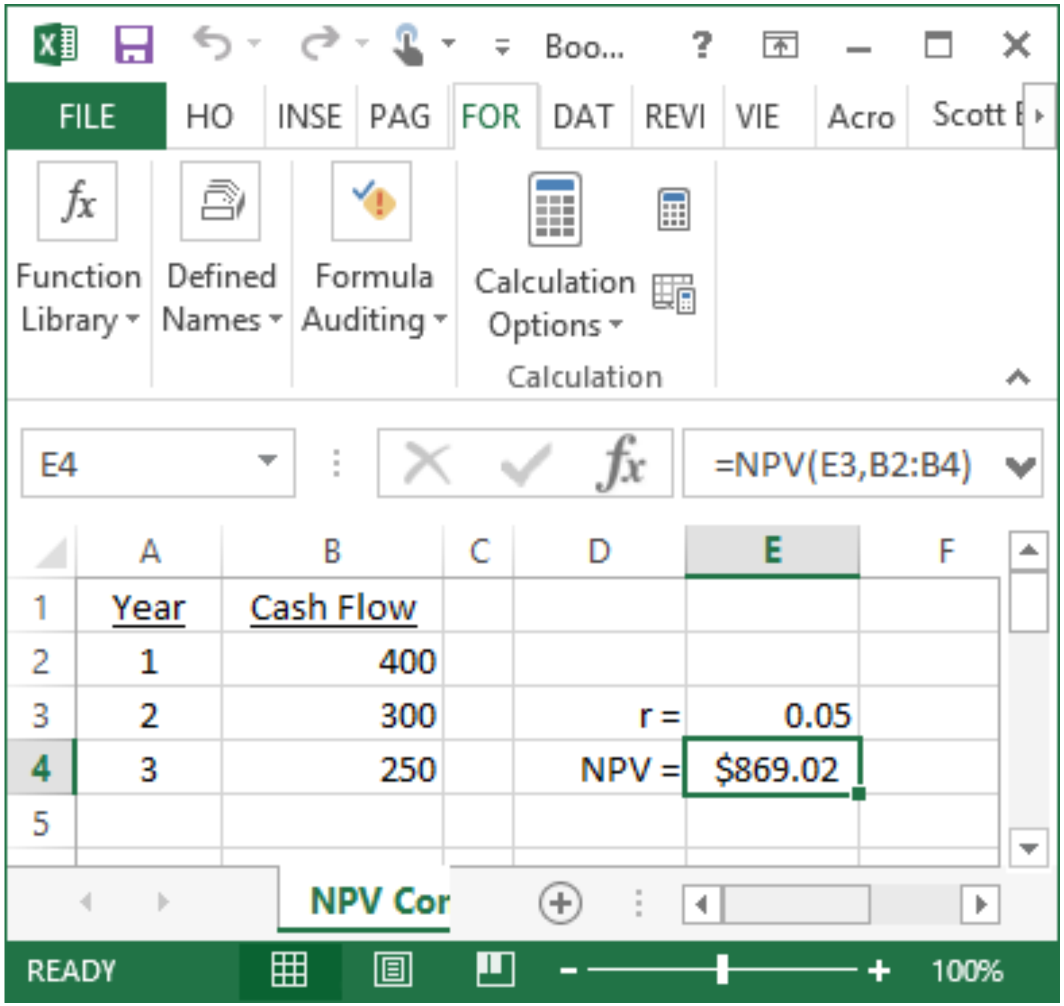
Financial Calculator Solution

- Input in "CF" register:
  - $CF_0 = 0$
  - $CF_1 = 400$
  - $CF_2 = 300$
  - $CF_3 = 250$
- Enter I = 5%, then press NPV button to get NPV = 869.02. (Here NPV = PV)

Spreadsheet Solution

- ✓ Setup the spreadsheet so that the cash flows are ordered sequentially
- ✓ Use the NPV function to solve for the present value of the non-constant cash flow series.

Spreadsheet Solution



Semiannual and Other Compounding Periods

- ✓ **Annual compounding** is the process of determining the future (present) value of a cash flow or series of cash flows when interest is added (computed) once per year.
- ✓ **Semiannual compounding** is the process of determining the future (present) value of a cash flow or series of cash flows when interest is added (computed) twice per year.

Compounding

- Will the FV of a lump sum be larger or smaller if we compound more often, holding the stated r constant?
  - If compounding is more frequent than once per year—for example, semiannually, quarterly, or daily—interest is earned on interest. Because interest is compounded more often, the future value will be larger.

Comparison of Different Interest Rates

r_{SIMPLE} = Simple (Quoted) Rate
Used to compute the interest paid each period

APR = Annual Percentage Rate = r_{SIMPLE}
APR is a non-compounded interest rate

EAR = Effective Annual Rate = r_{EAR}
The rate that would produce the same future value if annual compounding had been used

EAR for a simple rate of 10%, compounded semi-annually

$$EAR = r_{EAR} = \left(1 + \frac{r_{SIMPLE}}{m}\right)^{m} - 1$$

$$= \left(1 + \frac{0.10}{2}\right)^{2} - 1.0$$

$$= (1.05)^{2} - 1.0 = 0.1025 = 10.25\%$$

FV of $100 after 3 years if interest is 10% compounded semi-annually

$$FV_n = PV\left(1 + \frac{r_{SIMPLE}}{m}\right)^{m \times n}$$

interest is added (computed) twice per year

$$FV_{3\times2} = \$100\left(1 + \frac{0.10}{2}\right)^{2\times3}$$

$$= \$100(1.34010) = \$134.01$$

FV of $100 after 3 years if interest is 10% compounded quarterly

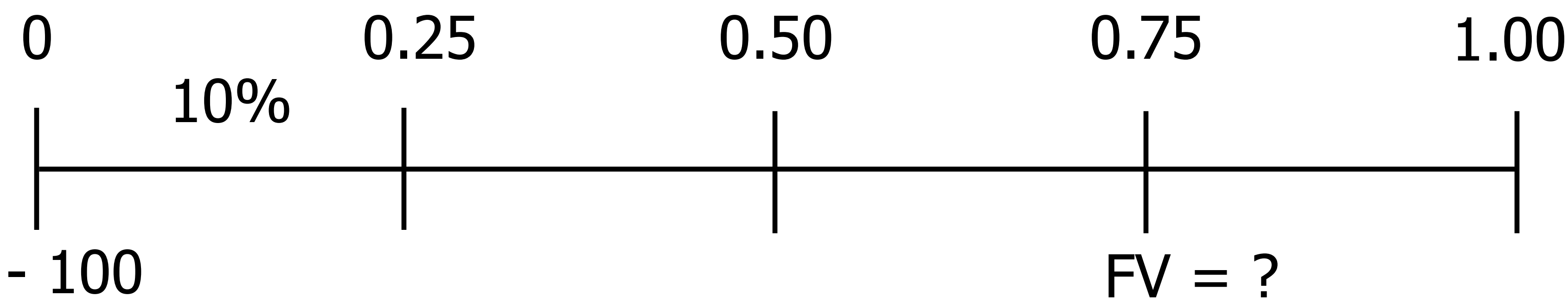
$$FV_n = PV\left(1 + \frac{r_{SIMPLE}}{m}\right)^{m \times n}$$

$$FV_{3\times 4} = \$100\left(1 + \frac{0.10}{4}\right)^{4\times 3}$$

$$= \$100(1.34489) = \$134.49$$

Fractional Time Periods

Example: $100 deposited in a bank at r_{EAR} = 10% for 0.75 of the year

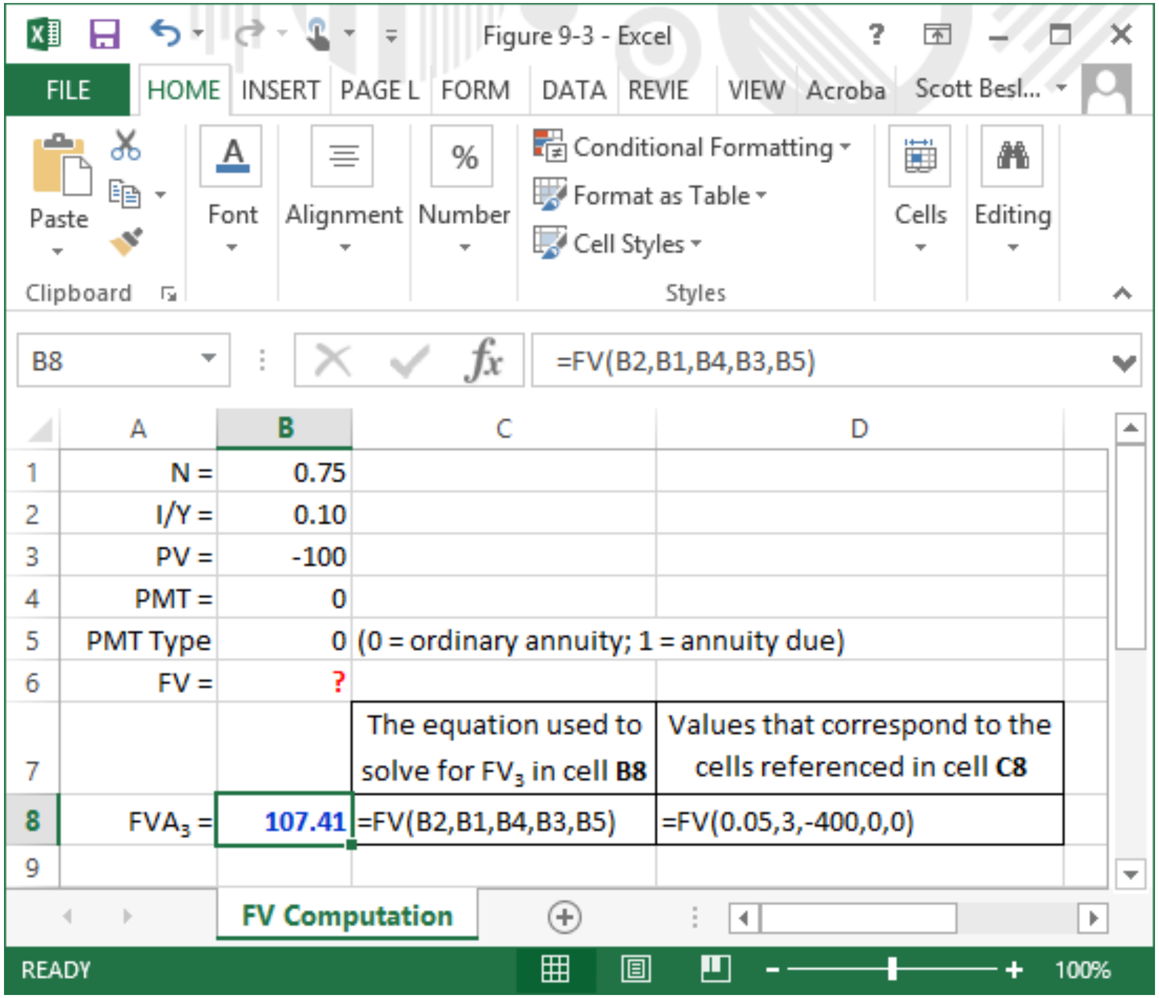


Financial Calculator Solution

Example: $100 deposited in a bank at EAR = 10% for 0.75 of the year

	N	I/Y	PV	PMT	FV
INPUTS	0.75	10	-100	0	?
OUTPUT					107.41

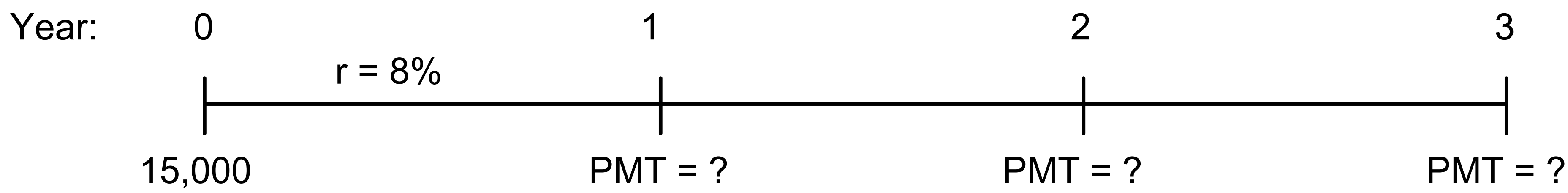
Spreadsheet Solution



Amortized Loans

- Amortized Loan: A loan that is repaid in equal payments over its life; payment includes both principal repayment and interest
- Amortization tables are widely used to determine how much of each payment represents principal repayment and how much represents interest.
- Financial calculators (and spreadsheets) can be used to set up amortization tables.

Amortization schedule for a $15,000, 8% loan that requires three equal annual payments



Step 1: Determine the Required Payments

	N	I/Y	PV	PMT	FV
INPUTS	3	8	15,000	?	0
OUTPUT				-5,820.50	

Step 2: Find Interest Charge for Year 1

$$INT_t = \text{Beginning balance}_t\,(r)$$

$$INT_1 = 15{,}000(0.08) = \$1{,}200.00$$

Step 3: Find Repayment of Principal in Year 1

Repayment	=	PMT - INT
	=	5,820.50 - $1200.00
	=	$4,620.50

Step 4: Find Ending Balance after Year 1

Ending bal. = Beginning bal. - Repayment
= $15,000 - 4,620.50 = $10,379.50

Repeat these steps for the remainder of the payments (Years 2 and 3 in this case) to complete the amortization table.

Loan Amortization Schedule
$15,000 Loan at 8 Percent Interest Rate

Year	Beg. of Year Balance (1)	Payment (2)	Interest @ 8% (3) = (1) x 0.08	Repayment of Principal (4) = (2) – (3)	End of Year Balance (5) = (1) – (4)
1	$15,000.00	$5,820.50	$1,200.00	$4,620.50	$10,379.50
2	10,379.50	5,820.50	830.36	4,990.14	5,389.36
3	5,389.36	5,820.50	431.15	5,389.35	0.01

The $0.01 remaining balance at the end of Year 3 results from a rounding difference.

Chapter Principles
Key Time Value of Money Concepts

- What are the three basic types of cash flow patterns?
  - Lump-sum amount – a single payment paid or received in the current period or some future period
  - Annuity - A series of equal payments that occur at equal time intervals
  - Uneven cash flow stream – multiple payments that are not equal

Chapter Principles
Key Time Value of Money Concepts

- How are dollars from different time periods compared when making financial decisions?
  - Dollars from different time periods must be stated in the same "Time Value" before they can be compared.
  - Dollars can be translated into the same time period by computing either present value or future value.

Chapter Principles
Key Time Value of Money Concepts

- ✓ How is the return on an investment determined?
  - ✓ The return is determined by the rate at which the investment grows over time.
  - ✓ Everything being equal, the current value of an investment is lower the higher the interest rate it earns in the future.

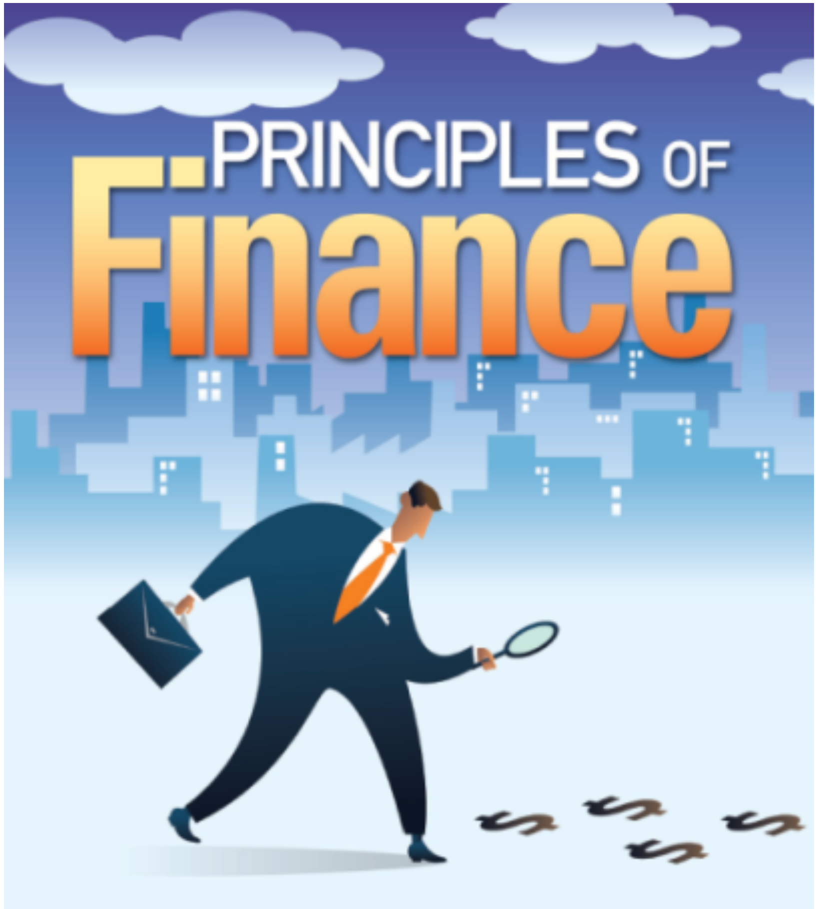
Chapter Principles
Key Time Value of Money Concepts

- What is the difference between the Annual Percentage Rate (APR) and the Effective Annual rate (EAR)?
  - APR is a simple noncompounded interest rate quoted on loans.
  - EAR is the actual interest (compounded) rate or rate of return.

Chapter Principles
Key Time Value of Money Concepts

- What is an amortized loan?
  - A loan paid off in equal payments over a specified period.
  - Each payment includes repayment of some principal and payment of interest.

End of Chapter 9



The Time Value of Money