## The Economics of Money, Banking, and Financial Markets

## Twelfth Edition



## Chapter 4

The Meaning of Interest Rates

## Preview

- Before we can go on with the study of money, banking, and financial markets, we must understand exactly what the phrase interest rates means. In this chapter, we see that a concept known as the yield to maturity is the most accurate measure of interest rate.


## Learning Objectives

- Calculate the present value of future cash flows and the yield to maturity on the four types of credit market instruments.
- Recognize the distinctions among yield to maturity, current yield, rate of return, and rate of capital gain.
- Interpret the distinction between real and nominal interest rates.


## Measuring Interest Rates

- Present value: a dollar paid to you one year from now is less valuable than a dollar paid to you today.
- Why: a dollar deposited today can earn interest and become $\$ 1 \times(1+\mathrm{i})$ one year from today.
- To understand the importance of this notion, consider the value of a $\$ 20$ million lottery payout today versus a payment of $\$ 1$ million per year for each of the next 20 years. Are these two values the same?


## Present Value

## Let $i=.10$

$$
\begin{gathered}
\text { In one year: } \$ 100 \times(1+0.10)=\$ 110 \\
\text { In two years: } \$ 110 \times(1+0.10)=\$ 121 \\
\text { or } \$ 100 \times(1+0.10)^{2}
\end{gathered}
$$

In three years: $\$ 121 \times(1+0.10)=\$ 133$

$$
\text { or } \$ 100 \times(1+0.10)^{3}
$$

In $n$ years
$\$ 100 \times(1+i)^{n}$

## Simple Present Value (1 of 2)

$$
\begin{gathered}
\mathrm{PV}=\text { today's (present) value } \\
\mathrm{CF}=\text { future cash flow (payment) } \\
i=\text { the interest rate } \\
\mathrm{PV}=\frac{\mathrm{CF}}{(1+i)^{n}}
\end{gathered}
$$

## Simple Present Value (2 of 2)

- Cannot directly compare payments scheduled in different points in the time line

|  |  |  |  |
| :---: | :---: | :---: | :---: |
| Year | $\$ 100$ | $\$ 100$ | $\$ 100$ |
| PV | 100 | 1 | 2 |

## Four Types of Credit Market Instruments

- Simple Loan
- Fixed Payment Loan
- Coupon Bond
- Discount Bond


## Yield to Maturity

- Yield to maturity: the interest rate that equates the present value of cash flow payments received from a debt instrument with its value today


## Yield to Maturity on a Simple Loan

$$
\begin{gathered}
\mathrm{PV}=\text { amount borrowed }=\$ 100 \\
\mathrm{CF}=\text { cash flow in one year }=\$ 110 \\
n=\text { number of years }=1 \\
\$ 100=\frac{\$ 110}{(1+i)^{1}} \\
(1+i) \$ 100=\$ 110 \\
(1+i)=\frac{\$ 110}{\$ 100} \\
i=0.10=10 \%
\end{gathered}
$$

For simple loans, the simple interest rate equals the yield to maturity

## Fixed-Payment Loan

The same cash flow payment every period throughout the life of the loan

LV = loan value
FP = fixed yearly payment
$n=$ number of years until maturity

$$
\mathrm{LV}=\frac{\mathrm{FP}}{1+i}+\frac{\mathrm{FP}}{(1+i)^{2}}+\frac{\mathrm{FP}}{(1+i)^{3}}+\ldots+\frac{\mathrm{FP}}{(1+i)^{n}}
$$

## Coupon Bond (1 of 4)

Using the same strategy used for the fixed-payment loan:

$$
\begin{gathered}
\mathrm{P}=\text { price of coupon bond } \\
\mathrm{C}=\text { yearly coupon payment } \\
\mathrm{F}=\text { face value of the bond } \\
n=\text { years to maturity date } \\
\mathrm{P}=\frac{\mathrm{C}}{1+i}+\frac{\mathrm{C}}{(1+i)^{2}}+\frac{\mathrm{C}}{(1+i)^{3}}+\ldots+\frac{\mathrm{C}}{(1+i)^{n}}+\frac{\mathrm{F}}{(1+i)^{n}}
\end{gathered}
$$

## Coupon Bond (2 of 4)

- When the coupon bond is priced at its face value, the yield to maturity equals the coupon rate.
- The price of a coupon bond and the yield to maturity are negatively related.
- The yield to maturity is greater than the coupon rate when the bond price is below its face value.


## Coupon Bond (3 of 4)

Table 1 Yields to Maturity on a 10\%-Coupon-Rate Bond Maturing in Ten Years (Face Value $=\$ 1,000$ )

| Price of Bond (\$) | Yield to Maturity (\%) |
| :---: | :---: |
| 1,200 | 7.13 |
| 1,100 | 8.48 |
| 1,000 | 10.00 |
| 900 | 11.75 |
| 800 | 13.81 |

## Coupon Bond (4 of 4)

- Consol or perpetuity: a bond with no maturity date that does not repay principal but pays fixed coupon payments forever

$$
P=\mathrm{C} / i_{c}
$$

$P_{c}=$ price of the consol
$C=$ yearly interest payment
$I_{\mathrm{c}}=$ yield to maturity of the consol
can rewrite above equation as this: $i_{c}=C / P_{c}$
For coupon bonds, this equation gives the current yield, an easy to calculate approximation to the yield to maturity

## Discount Bond

For any one year discount bond

$$
\begin{gathered}
i=\frac{\mathrm{F}-\mathrm{P}}{\mathrm{P}} \\
\mathrm{~F}=\text { Face value of the discount bond } \\
\mathrm{P}=\text { Current price of the discount bond }
\end{gathered}
$$

The yield to maturity equals the increase in price over the year divided by the initial price.

As with a coupon bond, the yield to maturity is negatively related to the current bond price.

## The Distinction Between Interest Rates and Returns (1 of 4)

- Rate of Return:

The payments to the owner plus the change in value expressed as a fraction of the purchase price

$$
\mathrm{RET}=\frac{\mathrm{C}}{\mathrm{P}_{t}}+\frac{\mathrm{P}_{t+1}-\mathrm{P}_{t}}{\mathrm{P}_{t}}
$$

RET $=$ return from holding the bond from time $t$ to time $t+1$

$$
\begin{gathered}
\mathrm{P}_{t}=\text { price of bond at time } t \\
\mathrm{P}_{t+1}=\text { price of the bond at time } t+1 \\
\mathrm{C}=\text { coupon payment } \\
\frac{\mathrm{C}}{\mathrm{P}_{t}}=\text { current yield }=i_{c} \\
\frac{\mathrm{P}_{t+1}-\mathrm{P}_{t}}{\mathrm{P}_{t}}=\text { rate of capital gain }=g
\end{gathered}
$$

## The Distinction Between Interest Rates and

## Returns (2 of 4)

- The return equals the yield to maturity only if the holding period equals the time to maturity.
- A rise in interest rates is associated with a fall in bond prices, resulting in a capital loss if time to maturity is longer than the holding period.
- The more distant a bond's maturity, the greater the size of the percentage price change associated with an interestrate change.
- Interest rates do not always have to be positive as evidenced by recent experience in Japan and several European states.


## The Distinction Between Interest Rates and Returns (3 of 4)

- The more distant a bond's maturity, the lower the rate of return the occurs as a result of an increase in the interest rate.
- Even if a bond has a substantial initial interest rate, its return can be negative if interest rates rise.


## The Distinction Between Interest Rates and Returns (4 of 4)

Table 2 One-Year Returns on Different-Maturity 10\%-CouponRate Bonds When Interest Rates Rise from 10\% to $20 \%$

| (1) <br> Years to Maturity <br> When Bond Is <br> Purchased | (2) <br> Initial <br> Current <br> Yield (\%) | (3) <br> Initial <br> Price <br> (\$) | (4) <br> Price <br> Next <br> Year* (\$) | (5) <br> Rate of <br> Capital Gain <br> (\%) | (6) <br> Rate of Return <br> [col (2) + col (5)] <br> (\%) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 30 | 10 | 1,000 | 503 | -49.7 | -39.7 |
| 20 | 10 | 1,000 | 516 | -48.4 | -38.4 |
| 10 | 10 | 1,000 | 597 | -40.3 | -30.3 |
| 5 | 10 | 1,000 | 741 | -25.9 | -15.9 |
| 2 | 10 | 1,000 | 917 | -8.3 | +1.7 |
| 1 | 10 | 1,000 | 1,000 | 0.0 | +10.0 |

*Calculated with a financial calculator, using Equation 3.

## Maturity and the Volatility of Bond Returns: Interest-Rate Risk

- Prices and returns for long-term bonds are more volatile than those for shorter-term bonds.
- There is no interest-rate risk for any bond whose time to maturity matches the holding period.


## The Distinction Between Real and Nominal Interest Rates

- Nominal interest rate makes no allowance for inflation.
- Real interest rate is adjusted for changes in price level so it more accurately reflects the cost of borrowing.
- Ex ante real interest rate is adjusted for expected changes in the price level
- Ex post real interest rate is adjusted for actual changes in the price level


## Fisher Equation

$$
\begin{gathered}
i=i_{r}+\pi^{e} \\
i=\text { nominal interest rate } \\
i_{r}=\text { real interest rate } \\
\pi^{e}=\text { expected inflation rate }
\end{gathered}
$$

When the real interest rate is low,
there are greater incentives to borrow and fewer incentives to lend.
The real interest rate is a better indicator of the incentives to borrow and lend.

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# Figure 1 Real and Nominal Interest Rates (Three-Month Treasury Bill), 1953-2017 



Sources: Nominal rates from Federal Reserve Bank of St. Louis FRED database: http://research.stlouisfed.org/fred2/. The real rate is constructed using the procedure outlined in Frederic S. Mishkin, "The Real Interest Rate: An Empirical Investigation," Carnegie-Rochester Conference Series on Public Policy 15 (1981): 151-200. This procedure involves estimating expected inflation as a function of past interest rates, inflation, and time trends, and then subtracting the expected inflation measure from the nominal interest rate.

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